vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam: Dynamic Econometrics

Code: E_EORM_DE

Examinator: Hande Karabiyik
Co-reader: Blasques Albergaria Amaral, F.

Date: March 20, 2017

Time: 15:15

Duration: 2 hours and 45 minutes

Calculator allowed: No

Graphical calculator allowed: No

Number of questions: 4 (where each question consists of multiple parts)

Type of questions: Open

Answer in: English

Remarks:

- All students should answer Question 1 and choose two questions out of Questions 2-4.
- Motivate all your answers.
- The students are allowed to take the exam questions with them. The questions do not have to be handed in after the exam

Credit score: 100 credits counts for a 10 from the exam, maximum credit score is 20 credits for the first question and 40 credits for the questions 2,3, 4.)

Grades: The grades will be made public before the 30th of March.

Inspection: For inspection you can make an appointment by email (h.karabiyik@vu.nl).

Number of pages: 12 (including front page)

Good luck!

Instructions

(i) All students should answer Question 1 and choose two questions out of Questions 2, 3, 4.

(ii) From Question 1, the students are required to answer 4 questions in total.

(iii) Please do not answer to more than necessary.

(iv) Answer the questions as detailed as possible. Use mathematical expressions when necessary. You can use words when you cannot provide a formal mathematical answer to the questions.

(v) If a question is not clear to you (because of a typo etc.), make your own assumptions to clarify the meaning of the question and then answer the question based on your assumptions.

Some standard results

Suppose that the scalar process $\{x_t\}$ follows the following data generating process:

$$x_t = x_{t-1} + u_t,$$

where $x_0 = 0$ and u_t has the following properties:

(a) $E(u_t) = 0$ for all t;

(b) σ^2 denotes the long run variance and σ^2 exists and $\sigma^2 > 0$;

(c) σ_u^2 denotes the contemporaneous variance and σ_u^2 exists and $\sigma_u^2 > 0$;

(d) $\sup_t E|u_t|^{\beta+\eta} < \infty$ for some $\eta > 0$ and $\beta > 2$;

(e) $\{u_t\}_1^\infty$ is strong mixing with mixing coefficients α_m that satisfy $\sum_1^\infty \alpha_m^{1-2/\beta} < \infty$, where the β is the same as in condition **(d)**.

Then the following results hold:

(1) $T^{-2}\sum_{t=1}^T x_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 [W(r)]^2 dr$;

(2) $T^{-1}\sum_{t=1}^T x_{t-1}u_t \xrightarrow{d} \frac{1}{2}\sigma^2 \left[W(1)^2 - \frac{\sigma_u^2}{\sigma^2}\right]$;

Question 1: True/False Questions (20 points out of 100 points)

Below you will find a list of **10** statements. All these are related to the concepts/topics/techqniques that have been discussed during the lectures. Some of these statements are correct, some are wrong, some need further clarification. Select **4** (not more than **4**) of these statements and provide a brief, to the point answer. In your answer you should first give a definition of the concepts mentioned in the statements. A formal answer using mathematics is possible, sometimes very useful but not always necessary.

(a) A unit root in the moving average part of an ARMA model implies nonstationarity.

(b) Granger causality and strong exogeneity conditions are statements about the underlying data generating process (DGP) while weak exogeneity is a statement about the parameter space of a model.

(c) Assume I have two I(2) and two I(1) processes. A situation where combinations of I(2) and I(1) variables my be I(0) can arise.

(d) Spurious regression means that we erroneously reject the null of no-cointegration.

(e) The concept of a white noise process and that a martingale difference sequence can be used interchangably.

(f) The identification of a cointegrating vector is only an issue when the cointegrating rank (the number of cointegrating vectors) is bigger than one.

(g) Under the assumption of cointegration, the "consistency" of the OLS estimator of a static cointegrating regression [between I(1) variables] of the type $y_t = \beta x_t + u_t$ requires independence between the "regressor" and the regression error.

(h) FMLS (Fully modified least squares) estimator is obtained by applying a correction to the regressor and to the static least squares estimator. This corrections are made to eliminate the second order bias of the static least squares estimator.

(i) Ergodicity of a process ensures that dependence vanishes asymptotically.

(j) "Nickell bias" refers to the bias of the pooled OLS estimator that arises when there is a correlation between the regressor and the error term in a static panel data model.

Question 2: Modeling, Integration Orders, Exogeneity (40 points out of 100 points)

1 - (1 - 2) = 2

1 - (1 + 2) = -2

Consider the following bivariate Data Generating Process for $\mathbf{w}_t = (y_t, x_t, z_t)'$:

$$\Delta y_t = \alpha_1(y_{t-1} - \beta_1 x_{t-1} - \beta_2 z_{t-1}) + \gamma_{11}\Delta y_{t-1} + \gamma_{12}\Delta x_{t-1} + \varepsilon_{1,t},$$
$$\Delta x_t = \alpha_2(y_{t-1} - \beta_1 x_{t-1} - \beta_2 z_{t-1}) + \gamma_{22}\Delta x_{t-1} + \gamma_{23}\Delta z_{t-1} + \varepsilon_{2,t},$$
$$\Delta z_t = \alpha_3(y_{t-1} - \beta_3 x_{t-1} - \beta_4 z_{t-1}) + \gamma_{31}\Delta y_{t-1} + \gamma_{33}\Delta z_{t-1} + \varepsilon_{3,t},$$

where we assume the following for the error terms:

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \sim IN\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix}\right]$$

for $t = 1, \ldots, T$.

(a) Discuss the nature of the series in terms of their (non-)stationarity, integration order and cointegration properties. In particular, establish and discuss the order of integration d, i.e. I(d), cointegration (as well as the number and the form of the cointegrating vectors, if any) in the following cases:

(i) $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $|\gamma_{11}| < 1$, $|\gamma_{22}| < 1$, $\gamma_{12} = \gamma_{23} = \gamma_{31} = 0$ and $\gamma_{33} = 1$;

(ii) $-2 < \alpha_1 < 0$, $\alpha_2 = \alpha_3 = 0$, $\gamma_{11} = \gamma_{12} = \gamma_{23} = \gamma_{31} = 0$, $|\gamma_{22}| < 1$ and $|\gamma_{33}| < 1$;

(iii) $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\alpha_3 \neq 0$ and $\gamma_{11} = \gamma_{12} = \gamma_{22} = \gamma_{23} = \gamma_{31} = \gamma_{33} = 0$.

(b) Under the restrictions: $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\gamma_{11} \neq 0$, $\gamma_{12} \neq 0$, $\gamma_{23} \neq 0$, $|\gamma_{22}| < 1$, derive the conditional error correction model (CECM) of y_t given x_t and its past. How would you estimate such a model? How would you test for no-cointegration? Do you need to make any additional assumptions about the system for more efficient estimation? Under what condition(s) x_t is weakly exogenous for the parameters of interest $\boldsymbol{\phi} = \{\alpha_1, \beta_1, \beta_2\}$?

(c) Under the restrictions: $\alpha_1 \neq 0$, $\alpha_2 = \alpha_3 = 0$, $|\gamma_{22}| < 1$, $|\gamma_{33}| < 1$, all other γ_{ij} are zero; show that $\boldsymbol{\beta}'\mathbf{w}_t$, where $\boldsymbol{\beta} = (1, -\beta_1, -\beta_2)'$ and $\mathbf{w}_t = (y_t, x_t, z_t)'$, is weakly stationary if $-2 < \alpha_1 < 0$.

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(d) Under the restrictions: $\alpha_1 \neq 0$, $\alpha_2 = \alpha_3 = 0$, $|\gamma_{22}| < 1$, $|\gamma_{33}| < 1$, all other γ_{ij} are zero; derive the vector moving average (VMA) representation for $\Delta \mathbf{w}_t = (\Delta y_t, \Delta x_t, \Delta z_t)'$ and show how you can determine the number of common stochastic trends.

Question 3: Asymptotic Derivations (40 points out of 100 points)

Consider the following Data Generating Process for the scalar process $\{y_t\}$:

$$y_t = y_{t-1} + u_t, \quad t = 1, \ldots, T; \quad y_0 = 0,$$
$$u_t = \epsilon_t + \theta \epsilon_{t-1},$$

where we assume that $\epsilon_t \sim i.i.d(0, \sigma^2)$.

(a) Derive the long-run and contemporaneous variances of u_t.

(b) Using your findings from (a), as well as standard unit root limiting results, derive the asymptotic distribution of $T(\widehat{\rho}_{LS} - 1)$ in terms of θ, σ^2 and appropriate functions and functionals of Brownian motions, where

$$\widehat{\rho}_{LS} = \left(\sum_{t=1}^{T} y_{t-1}^2 \right)^{-1} \left(\sum_{t=1}^{T} y_t y_{t-1} \right).$$

(c) Now consider the Instrumental Variable (IV) type estimator, where y_{t-2} is used as an instrument, which can be written as

$$\widehat{\rho}_{IV} = \left(\sum_{t=1}^{T} y_{t-1} y_{t-2} \right)^{-1} \left(\sum_{t=1}^{T} y_t y_{t-2} \right).$$

Using your findings from (a), as well as standard unit root limiting results, derive the asymptotic distribution of $T(\widehat{\rho}_{IV} - 1)$ in terms of θ, σ^2 and appropriate functions and functionals of Brownian motions.

Hint: Note that you can use the fact that $y_{t-j} = y_{t-1} - u_{t-1} - u_{t-2} - \ldots - u_{t-j-1}$.

(d) Comment on the asymptotic properties of $\widehat{\rho}_{LS}$ and $\widehat{\rho}_{IV}$. Compare their asymptotic distributions. Discuss which one should be preferred over the other and why.

(e) The t–statistic for a unit root test based on the IV estimator can be written as

$$t_{\widehat{\rho}_{IV}=1} = \left(\sum_{t=1}^{T} y_{t-1} y_{t-2} \right)^{1/2} (\widehat{\rho}_{IV} - 1)\hat{\sigma}^{-1},$$

where $\hat{\sigma}$ is an estimate of the long run variance that assumes the exact knowledge of the MA structure. Is the t–test $t_{\widehat{\rho}_{IV}=1}$ consistent? Explain.

Hint: Use the previous results as well as the little and big $O_p(.)$ notation.

Question 4: Empirical Application (40 points out of 100 points)

(a) Econometricians A and B want to analyze money demand in Eyesland using a century long annual economic time series for the period 1900 – 2000. They use the following variables: the log of the money stock M2 (denoted by m_t, the log of the price index (p_t), the log of real GDP (y_t) and the central bank discount rate (R_t).

(i) Econometrician A estimates a potential (static) long-run money demand function by OLS and it yields the following results (standard errors are reported in parentheses):

$$\widehat{(m-p)}_t = \underset{(0.02)}{1.103}\ y_t - \underset{(0.24)}{0.57}\ R_t,$$

with a $R^2 = 0.98$. Given these results he concludes that the long run elasticities are highly significant and the income elasticity is clearly significantly different from 1. You are asked to interpret the results reported above and comment on the appropriateness of his analysis. Is there any evidence in favor of the existence of a long run relation? Could you use this static regression to test the null hypothesis of unit income elasticity analysis? If not what would you propose?

(ii) Econometrician B, first estimates the single equation CECM and obtains the following results:

$$\begin{aligned}\Delta\widehat{(m-p)}_t &= \underset{(0.14)}{0.33}\ \Delta y_t - \underset{(0.04)}{0.06}\ (m-p)_{t-1} + \underset{(0.02)}{0.03}\ y_{t-1} \\ &\quad - \underset{(0.11)}{0.01}\ R_{t-1} - \underset{(0.26)}{0.32}\ \Delta R_t,\end{aligned}$$

with a $R^2 = 0.64$. He then applies the Johansen's MLE approach to the VECM that yields the results reported in the Table below.

Hypothesized rank	Trace statistic	0.05 critical value	Probability
Zero	39.26	47.85	0.25
At most one	15.45	29.79	0.75
At most two	4.41	15.49	0.86
At most three	0.38	3.84	0.53

You are asked to explain reasons behind his choices of tests and estimations and to help the econometrician to make a conclusion about the long run money demand. Also, explain whether these CECM and VECM results are in accordance or in conflict with the static regression results reported above? Explain the situation this econometrician is facing. Motivate all your answers.

(b) Econometrician C is analyzing a panel data set on annual house prices in 245 districts in Nicetricht over 4 years. Previous research shows that house prices are dynamic in nature that is current prices are functions of past prices. Furthermore, it is highly suspected that an individual specific effect is present in the model that represents the heterogeneity among districts. Econometrician C realizes these and assumes the following model, where he denotes the variable by $p_{i,t}$:

$$p_{i,t} = \mu_i + \lambda p_{i,t-1} + \varepsilon_{i,t},$$

where μ_i is the individual specific effect for district i and $\varepsilon_{i,t}$ is the error term for district i at time t. Econometrician C is interested in estimating λ. One estimator that can be used to estimate this model is the "fixed effects estimator", denoted by $\widehat{\lambda}_{FE}$, which is a pooled OLS estimator that uses demeaned variables. He knows that using the "fixed effects" estimator might not be a good idea. He looks into the literature and discovers the existence of two more methods, namely "Anderson-Hsiao estimator" and "Arellano-Bond estimator".

(i) Write down the formula for the "fixed effects estimator". Motivate the use of $\widehat{\lambda}_{FE}$ as an estimator. Show mathematically that the within transformation (time demeaning) eliminates the individual fixed effects.

(ii) He finds that $\widehat{\lambda}_{FE} = 0.5$. Explain why this result is not reliable. What is the problem with this estimator? When does it occur? Why does it occur? Explain in detail.

(iii) Explain and discuss the methodologies of the 'Anderson-Hsiao estimator" and "Arellano-Bond estimator". Discuss whether using these estimators solves the problem that is faced while using the fixed effects estimator.