vrije Universiteit *amsterdam*

Faculty of Economics and Business Administration

Exam:	Dynamic Econometrics
Code:	E_EORM_DE
Examinator:	Hande Karabiyik
Co-reader:	Blasques Albergaria Amaral, F.
Date:	December 19, 2016
Time:	08:30
Duration:	2 hours and 45 minutes
Calculator allowed:	No
Graphical calculator allowed:	No
Number of questions:	4 (where each question consists of multiple parts)
Type of questions:	Open
Answer in:	English

Remarks:

- All students should answer Question 1 and choose two questions out of Questions 2-4.
- Motivate all your answers.
- The students are allowed to take the exam questions with them. The questions do not have to be handed in after the exam

Credit score:	100 credits counts for a 10 from the exam, maximum credit score is 20 credits for the first question and 40 credits for the questions 2,3, 4.)
Grades:	The grades will be made public before the 16th of January.
Inspection:	For inspection you can make an appointment by email (h.karabiyik@vu.nl).
Number of pages:	9 (including front page)

Good luck!

Instructions

(i) All students should answer Question 1 and choose two questions out of Questions 2, 3, 4.

(ii) From Question 1, the students are required to answer 4 questions in total.

(iii) Please do not answer to more than necessary.

(iv) Answer the questions as detailed as possible. Use mathematical expressions when necessary. You can use words when you cannot provide a formal mathematical answer to the questions.

(v) If a question is not clear to you (because of a typo etc.), make your own assumptions to clarify the meaning of the question and then answer the question based on your assumptions.

Some standard results

Suppose that the scalar process $\{x_t\}$ follows the following data generating process:

$$x_t = x_{t-1} + u_t,$$

where $x_0 = 0$ and u_t has the following properties:

(a) $E(u_t) = 0$ for all t;

(b) σ^2 denotes the long run variance and σ^2 exists and $\sigma^2 > 0$;

(c) σ_u^2 denotes the contemporaneous variance and σ_u^2 exists and $\sigma_u^2 > 0$;

(d) $\sup_t E|u_t|^{\beta+\eta} < \infty$ for some $\eta > 0$ and $\beta > 2$;

(e) $\{u_t\}_1^\infty$ is strong mixing with mixing coefficients α_m that satisfy $\sum_1^\infty \alpha_m^{1-2/\beta} < \infty$, where the β is the same as in condition **(d)**.

Then the following results hold:

(1) $T^{-2}\sum_{t=1}^T x_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 [W(r)]^2 dr$;

(2) $T^{-1}\sum_{t=1}^T x_{t-1}u_t \xrightarrow{d} \frac{1}{2}\sigma^2 \left[W(1)^2 - \frac{\sigma_u^2}{\sigma^2}\right]$;

Question 1: True/False Questions (20 points out of 100 points)

Below you will find a list of **10** statements. All these are related to the concepts/topics/techqniques that have been discussed during the lectures. Some of these statements are correct, some are wrong, some need further clarification. Select **4** (not more than **4**) of these statements and provide a brief, to the point answer. A formal answer using mathematics is possible, sometimes very useful but not always necessary.

(a) The data generating process for a stochastic process x_t is given by the econometric model of this process.

(b) A process that is strictly stationary is automatically weakly (wide-sense) stationary. The opposite is not true.

(c) The property of martingale difference sequence is weaker than independence and also weaker than uncorrelatedness.

(d) The process, $x_t = x_{t-1} + u_t$, where $u_t \sim i.i.d.(0, \sigma^2)$ is nonstationary because its mean and variance are tending to infinity as $T \to \infty$.

(e) The OLS estimator of the AR coefficient of a unit root AR(1) process yields has a asymptotic distribution that is independent of nuisance parameters.

(f) The identification of a cointegrating vector is only an issue when the cointegrating rank (the number of cointegrating vectors) is bigger than one.

(g) Under the assumption of cointegration, the "consistency" of the OLS estimator of a static cointegrating regression [between I(1) variables] of the type $y_t = \beta x_t + u_t$ requires independence between the "regressor" and the regression error.

(h) FMLS (Fully modified least squares) estimator is obtained by applying a correction to the regressor and to the static least squares estimator. This corrections are made to eliminate the second order bias of the static least squares estimator.

(i) "Trace test" can be used to determine the cointegration rank of a system.

(j) "Fixed effects estimator" is an estimator that is used to estimate panel data models where there is an unobserved individual specific time-fixed component in the error term.

Question 2: Modeling, Integration Orders, Exogeneity (40 points out of 100 points)

Consider the following bivariate Data Generating Process for $\mathbf{w}_t = (y_t, z_t)'$:

$$\Delta y_t = \alpha_1(y_{t-1} - \beta_1 z_{t-1}) + \gamma_{11}\Delta y_{t-1} + \gamma_{12}\Delta z_{t-1} + \varepsilon_{1,t};$$
$$\Delta z_t = \alpha_2(y_{t-1} - \beta_1 z_{t-1}) + \alpha_3 z_{t-1} + \gamma_{21}\Delta y_{t-1} + \gamma_{22}\Delta z_{t-1} + \varepsilon_{2,t},$$

where $\beta_1 \neq 0$ and

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim IN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right]$$

for $t = 1, \ldots, T$.

(a) Discuss the nature of the series in terms of their (non-)stationarity, integration order and cointegration properties. In particular, establish and discuss the order of integration d, i.e. I(d), cointegration (as well as the number and the form of the cointegrating vectors, if any) in the following cases:

- **(i)** $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\gamma_{12} \neq 0$, $\gamma_{21} = 0$, $\gamma_{11} = 0$ and $\gamma_{22} = 1$;
- **(ii)** $-2 < \alpha_1 < 0$, $\alpha_2 = \alpha_3 = 0$, $\gamma_{12} = \gamma_{21} = \gamma_{11} = 0$ and $|\gamma_{22}| < 1$;
- **(iii)** $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\alpha_3 \neq 0$, $\gamma_{11} = \gamma_{22} = \gamma_{12} = 0$, $\gamma_{21} \neq 0$.

(b) Under the restrictions: $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\alpha_3 = 0$, $\gamma_{11} \neq 0$, $\gamma_{12} \neq 0$, $\gamma_{21} = 0$, $|\gamma_{22}| < 1$, derive the conditional error correction model (CECM) of y_t given z_t and the past. How would you estimate such a model? How would you test for no-cointegration? Do you need to make any additional assumptions about the system for more efficient estimation? Under what condition(s) z_t is weakly exogenous for the parameters of interest $\boldsymbol{\phi} = \{\alpha_1, \beta_1\}$?

(c) Under the restrictions: $\alpha_1 \neq 0$, $\alpha_2 = \alpha_3 = 0$, $|\gamma_{22}| < 1$, $\gamma_{11} = \gamma_{12} = \gamma_{21} = 0$; show that $\boldsymbol{\beta}'\mathbf{w}_t$, where $\boldsymbol{\beta} = (1, -\beta_1)'$ and $\mathbf{w}_t = (y_t, z_t)'$, is weakly stationary if $-2 < \alpha_1 < 0$.

Question 3: Asymptotic Derivations (40 points out of 100 points)

Consider the following Data Generating Process for the scalar process $\{y_t\}$:

$$y_t = y_{t-1} + u_t, \quad t = 1, \dots, T; \quad y_0 = 0,$$
$$u_t = \epsilon_t + \theta\epsilon_{t-1},$$

where we assume that $\epsilon_t \sim i.i.d(0, \sigma^2)$.

(a) Derive the long-run and contemporaneous variances of u_t.

Hint: Note that the long-run variane of a process, u_t, is equal to $\lim_{T\to\infty} E[T^{-1}(\sum_{t=1}^{T} u_t)^2]$ and the contemporaneous variance is $\lim_{T\to\infty} E[u_t^2]$.

(b) Using your findings from **(i)**, as well as standard unit root limiting results, derive the asymptotic distribution of $T(\widehat{\rho}_{LS} - 1)$ in terms of θ, σ^2 and appropriate functions and functionals of Brownian motions, where

$$\widehat{\rho}_{LS} = \left(\sum_{t=1}^{T} y_{t-1}^2\right)^{-1}\left(\sum_{t=1}^{T} y_t y_{t-1}\right).$$

(c) Now consider the Instrumental Variable (IV) type estimator, where y_{t-2} is used as an instrument, which can be written as

$$\widehat{\rho}_{IV} = \left(\sum_{t=1}^{T} y_{t-1}y_{t-2}\right)^{-1}\left(\sum_{t=1}^{T} y_t y_{t-2}\right).$$

Using your findings from **(i)**, as well as standard unit root limiting results, derive the asymptotic distribution of $T(\widehat{\rho}_{IV} - 1)$ in terms of θ, σ^2 and appropriate functions and functionals of Brownian motions.

Hint: Note that you can use the fact that $y_{t-j} = y_{t-1} - u_{t-1} - u_{t-2} - \ldots - u_{t-j-1}$.

(d) Comment on the asymptotic properties of $\widehat{\rho}_{LS}$ and $\widehat{\rho}_{IV}$. Compare their asymptotic distributions. Which estimator is more appropriate when we want to test for $\mathcal{H}_0 : \quad \rho = 1$? Explain how we can test for this hypothesis (i.e. which critical values we need to use)?

Question 4: Empirical Application (40 points out of 100 points)

(a) Econometrician A is analyzing a data set that consists of two variables over the period 1861 – 1988, namely the logarithm of the real money demand (m_t) and the logarithm of the real GDP (y_t) of the Neverlands. This data set is known to be non-stationary. Econometrician A expects m_t to be cointegrated with y_t and also he is interested in whether income elasticity of real money demand is equal to unity or not. He estimates by OLS a static regression of m_t on a constant and on y_t and obtains the following results (asymptotic standard errors are in parentheses):

$$\widehat{m}_t = \underset{(0.56)}{-2.23} + \underset{(0.03)}{0.88}\ y_t,$$

where $R^2 = 0.90$. By computing the $t-$statistic for the null hypothesis of unit income elasticity of money demand, he concludes that although the income elasticity is significantly smaller than one, the relation between real money demand and income is significant.

(i) Explain the situation faced by Econometrician A. What are the possible situations that might occur?

(ii) Comment on the conclusion of Econometrician A. Is the analysis of Econometrician A complete? If not, discuss in detail what else he needs to do to complete his analysis.

Hint: For instance, if he needs to conduct a certain test, describe the testing procedure, the asymptotic properties of the test statistic, critical values that should be used etc.

(b) Econometrician B is analyzing a panel data set on quarterly consumption of 211 families over 4 quarters. Previous research shows that household consumption data is dynamic in nature due to the presence of the lagged version of this variables in its model and it is highly suspected that an individual specific effect is present in the error term of the model that represents the heterogeneity among the households. Econometrician B realizes these and assumes the following model, where he denotes the variable by $c_{i,t}$:

$$c_{i,t} = \lambda c_{i,t-1} + u_{i,t},$$

where $u_{i,t} = \mu_i + \varepsilon_{i,t}$. Then she regresses $\widetilde{c}_{i,t} = c_{i,t} - T^{-1}\sum_{t=1}^{T} c_{i,t}$ on $\widetilde{c}_{i,t-1} = c_{i,t-1} - \sum_{t=1}^{T} c_{i,t-1}$ and obtains the pooled OLS estimator as

$$\widehat{\lambda}_{FE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \widetilde{c}_{i,t}\widetilde{c}_{i,t-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T} \widetilde{c}_{i,t-1}^{2}}.$$

She finds that $\widehat{\lambda}_{FE} = 0.8$. She thinks that she can rely on this result to say something reliable about the dimension of the dynamic nature of household consumption.

(i) Describe the situation faced by Econometrician A. Motivate the use of $\widehat{\lambda}_{FE}$ as an estimator. Show mathematically that the within transformation (time demeaning) eliminates the individual fixed effects.

(ii) We know that there is a problem with this estimator. What is the problem? When does it occur? Why does it occur? Explain in detail.

(iii) Propose and discuss an alternative estimator that is more proper/optimal to be used in this setup.