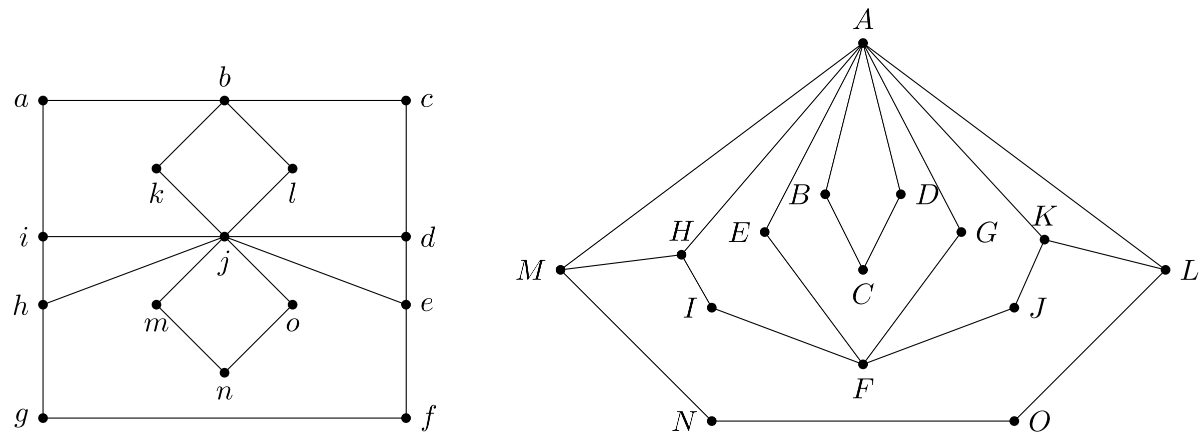
Please justify your answers! Even a correct answer without full explanation scores badly.

The use of books, lecture notes, calculators, etc. is not allowed.

Question 1. Consider the graphs G (below left) and H (below right) given in the following figure.



(a) Does G contain an Eulerian trail? Either give an Eulerian trail in G, or prove that such a trail does not exist.

(b) Is G bipartite? Either give a bipartite partition of the vertices of G or show that such a partition does not exist.

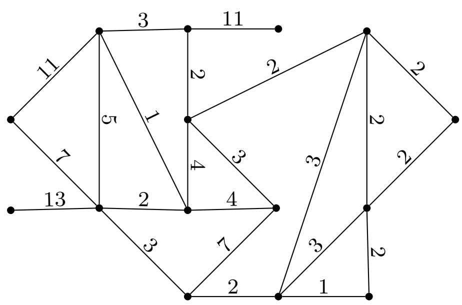
(c) Determine the chromatic number $\chi(G)$ of G. Prove your assertion.

(d) Find an edge cover of G consisting of seven vertices.

(e) Determine whether G and H are isomorphic graphs. Prove your assertion.

Question 2. (a) Draw a labelled tree with Prüfer sequence $3, 4, 2, 2, 2, 2, 3$. Describe your steps.

(b) Use Kruskal's algorithm to determine a minimum weight spanning tree of the weighted graph given in the figure below, and compute the total weight of that tree. Describe your steps.



Exam continues on page 2!

Question 3. Let G be a connected planar bipartite graph with n vertices, q edges, and r regions/faces (in any planar representation). Suppose G does not contain any cycle of length smaller than 7 and has a region bounded by 10 edges and a region bounded by 12 edges.

(a) Show that $q \geq 4r + 3$.

(b) Show that $4n \geq 3q + 11$.

`Note: You may use the inequality given in (a) above for answering (b).`

Question 4. Let G be a graph of order n. Suppose G has q edges and c connected components. Prove that $q + c \geq n$.

Hint: Assume first G is connected and consider a spanning tree.

Question 5. Suppose G is a 4-regular graph. Prove that we can colour the edges of G using two colours such that every vertex is incident with two edges of each colour.

Hint: Is G Eulerian?

Maximum score per subitem

1a: 4	2a: 8	3a:12	4: 18	5: 16
1b: 4	2b:10	3b:6		
1c: 4				
1d: 4				
1e: 4				

Maximum Total = 90

Mark = 1 + (Total/10)