Midterm Exam | **XB_0008: Discrete Mathematics**
VU Amsterdam | **23 November 2020 (12:00-14:00)**

Please justify your answers! Even a correct answer without full explanation scores badly.

Your hand-written answers must be submitted through Canvas in a single PDF file within 10 minutes after completing the exam.

The use of books, lecture notes, calculators, etc. is not allowed.

Question 1. Consider the graphs G and H given in Figure 1 below.

(a) Does G contain an Eulerian trail? Either give an Eulerian trail in G, or prove that such a trail does not exist.

(b) Are G and H isomorphic graphs? Prove your assertion.

(c) Is G planar? Either give a planar representation of G, or prove that such a representation does not exist.

(d) Determine the chromatic number of G. Prove your assertion.

(e) Does G contain any perfect matching? Either give a perfect matching of G or prove that such a matching does not exist.

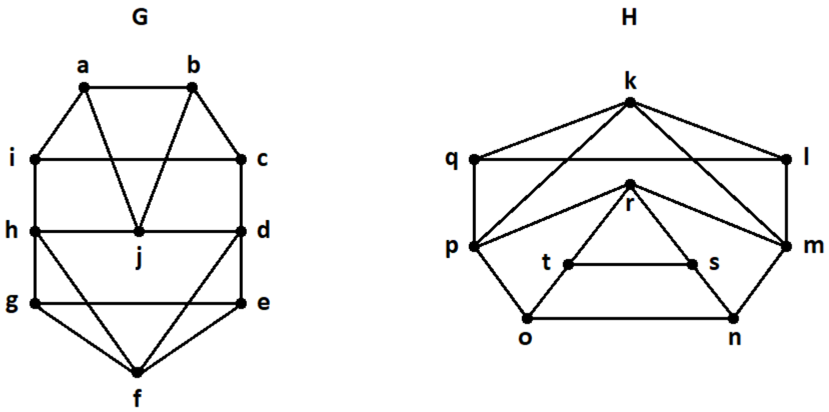

FIGURE 1

Question 2. Let G be a graph. Suppose G has a vertex v of odd degree. Show that there is a path from v to another vertex with odd degree.

Question 3. Let G be a graph of order $n \geq 3$ with vertices labelled $v_1, v_2, \ldots, v_n$ and A be the adjacency matrix of G. Suppose the minimum degree of G is at least $n/2$; i.e. $\delta(G) \geq n/2$.

Show that for every i there is a j, where $1 \leq i, j \leq n$ such that $[A]_{i,j} \neq 0$ and $[A^{n-1}]_{i,j} \neq 0$.

Exam continues on page 2!

Question 4. (a) Draw a labelled tree with Prüfer sequence $3, 2, 2, 5, 7, 2, 5$. Describe your steps.

(b) Use Kruskal's algorithm to determine a minimum weight spanning tree of the weighted graph given in Figure 2 below, and compute the total weight of that tree. Describe your steps.

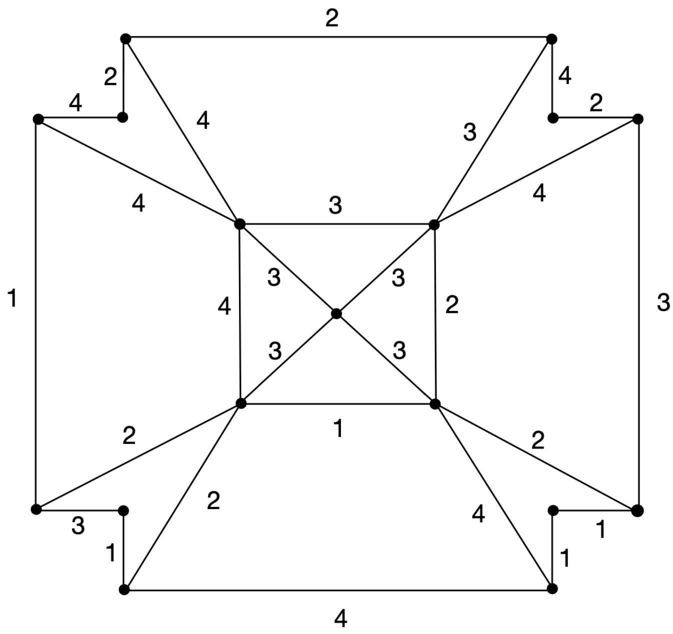

FIGURE 2

Question 5. Let G be a connected planar graph with $n \geq 6$ vertices, q edges, and r regions/faces (in any planar representation). Suppose G does not contain any cycle of length 3 and has a region bounded by 6 edges.

(a) Show that $q \geq 2r + 1$.

(b) Show that $2n \geq q + 5$.

`Note: You may use the inequality given in (a) above for answering (b).`

Maximum score per subitem

1a: 4 1b: 4 1c: 4 1d: 4 1e: 2	2: 14	3: 20	4a: 8 4b: 10	5a: 14 5b: 6

Maximum Total = 90

Mark = 1 + (Total/10)