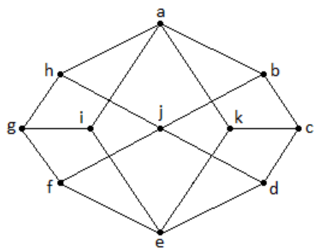
Practice Midterm	**XB-0008: Discrete Mathematics**
VU Amsterdam	**2019 Autumn (time: 2 hours)**

- Correct answers without full explanation score badly, so justify your steps.
- The use of books, lecture notes, calculators, etc. is *not* allowed.

(1) Consider the following graph G:



(a) Is G bipartite? Either give the bipartite sets, or prove that such a partition does not exist.
(b) Is G Hamiltonian? Either give a Hamiltonian cycle, or prove that it does not exist.
(c) Is G planar? Either give a planar representation, or prove that it does not exist.
(d) Determine the maximum number of edges in a matching in G. (As always, explain/verify your assertion.)

(2) Let G be a graph of order n with less than $n(n-1)/2$ edges. Show that G is $(n-1)$-colorable.

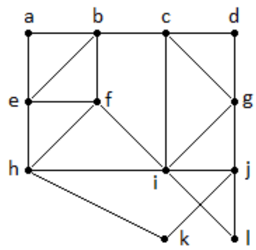
(3) Let G be a graph with adjacency matrix A. show that:

$$G \text{ is bipartite } \Leftrightarrow \text{ for all positive odd integers } k \text{ we have } \mathrm{Tr}(A^k) = 0.$$

(4) Let G be a k-regular Eulerian graph of even order. Prove that G has an even number of edges.

(5) Draw a labeled tree whose Prüfer sequence is $1, 3, 1, 5, 1, 7$. Describe your steps.

(6) Use Hierholzer's algorithm to find an Eulerian circuit in the following graph. Use $R_1 : a, b, c, g, i, h, e, a$ as your initial circuit. Describe your steps.



(7) Let G be a connected planar graph that is not a tree, with v vertices, e edges, and f regions/faces (in a planar representation). Suppose that G has at least m vertices of degree 1.
(a) Show that $2e \geq 3f + 2m$.
(Hint: consider $\sum_R (b(R) + 2c(R))$, where $c(R)$ denotes the number of edges that come into contact with region R but not with another region.)
(b) Show that $3(v-2) \geq e + 2m$.
(Note: you are allowed to use the inequality given at (a) above for answering (b).)