VU UNIVERSITY AMSTERDAM

Faculty of Economics and Business Administration

Derivatives 4.4

– How to Prepare for and handle the Exam –

Norman Seeger

How to prepare for the exam:

- Main material to prepare for the exam are the lecture slides. Use the Hull book to read up on things which are not clear to you in the slides.
- Topics in the slides I have been repeatedly discussing and gave more attention during class are more important to me and, therefore, are also more important for the exam.
- The VBA tutorial problems are equally important for the exam. Maker sure you can solve all assignment questions on your own. Also understand your own VBA solutions.
- VBA tutorial problems are designed in a way to also strengthen your theoretical understanding of the material. So working on those is a win-win :-) .
- Solve the additional exercises provided to you below.
- Make your own exercises. Think from my point of view what might be interesting to ask you.

Structure of exam:

- Provide your name and student id at the beginning of the exam
- The exam will consist of a majority open questions for which the answers need to be typed in word.
- The solutions have to be written down in the space indicated by the boxes. Write only in the boxes! The provided space gives you and indication for how much space you need to write down the solution I expect from you.
- There will be also some multiple choice questions for which the answers need to be filled in in an excel sheet.

- There will be a good part of programming problems (roughly 1/3 of the exam) that need to be solved by programming VBA. The numerical results of the programs need to be transferred to an excel sheet and will be graded.
- With all questions there comes an indication of how many points you can earn for the total question and how much each subquestion is worth. You can interpret the points also in way of how much time to spend for solving a question. One point is one minute of time. If one subquestion is worth 5 points you should spend roughly 5 minutes to solve the question.
- At the end of the exam there is a formulary. Check out what is provide to you. The formulary is not provided to you before the exam. If a big formula is needed to solve a question I will provide it in the formulary. e.g. Black-Scholes formula. Short standard formulas such as q probabilities in the CRR model you have to know by heart.
- Use of a graphical calculator is not allowed. You have excel at your disposal during the exam; the best graphical calculator of the world.

Handling the exam:

- Scan the exam first, to check which are the questions that are most easy to solve for you. Start with those in order to collect as much points as possible
- Read the questions carefully; think for a moment about what is the best way to solve the question. Often there are multiple ways of solving a question but one way might be smarter than another.
- If you cannot solve a subquestion go on to the next one and come back later when you solved all the rest. Collect points!
- Sometimes there is more information than needed

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– Additional Exercises –

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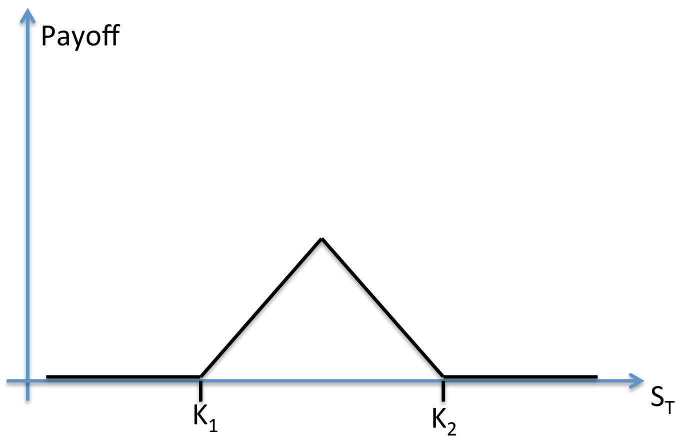
Part I

Old Exam Exercises

1. Structuring and Replication **(10 Points)**

Consider the following payoff profile.



(a) Give one portfolio of securities that replicates this payoff profile. For your replicating portfolio you are allowed to choose out of the following securities. All derivatives are written on the same underlying S and have the same time to maturity T. One can invest in long and short positions. (6 points)

- Underliyng S; $S_0 = 100$
- Bond (notional $\frac{K_1+K_2}{2}$); $B_0 = 90.91$
- Forward(K_1); $F_0(K_1) = 0$
- Call(K_1); $C_0(K_1) = 1.20$
- Call(K_2); $C_0(K_2) = 0.02$
- Call($\frac{K_1+K_2}{2}$); $C_0(\frac{K_1+K_2}{2}) = 0.34$
- Put(K_1); $P_0(K_1) = 0.01$
- Put(K_2); $P_0(K_2) = 0.82$
- Put($\frac{K_1+K_2}{2}$); $P_0(\frac{K_1+K_2}{2}) = 0.14$

(b) What will be the price of the replicating portfolio of the above given payoff profile. (4 points)

2. Forward Contract **(10 Points)**

Assume that the current ($t = 0$) stock price equals 40 and the stock pays no dividends. The risk-free interest rate equals 10% (continuous compounding). Consider a forward contract on this stock that expires in one year ($t = 1$).

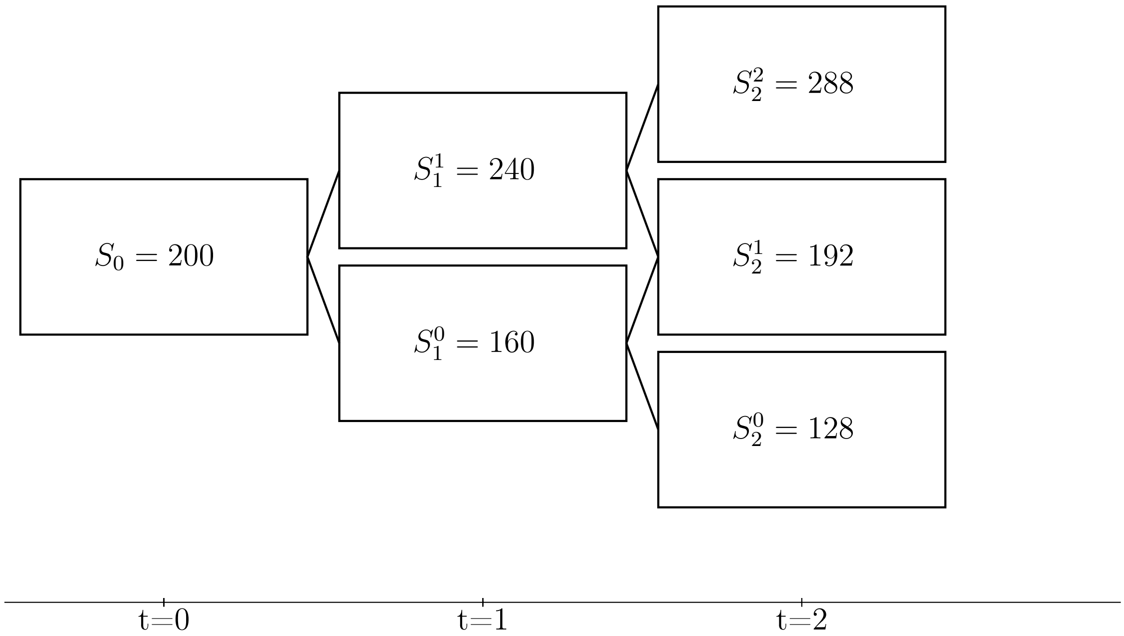
(a) Determine the forward price, F_0, and the forward value, FV_0, at time $t = 0$. (4 points)

(b) During the next six months the stock price rises to 45. Determine the forward price, $F_{0.5}$, and the forward value, $FV_{0.5}$, in six months, i.e. at time $t = 0.5$. (6 points)

3. Binomial Model **(30 Points)**

Consider the following two-period binomial tree for a non-dividend paying stock. The length of one period is one year. The risk-free rate is 5% (continuous compounding) and the current stock price is $S_0 = 200$.

Round all your factors to three decimal places.



(a) Determine the price of a European put with exercise price $K = 200$ that expires in two years ($t = 2$) using risk-neutral valuation. (8 points)

(b) Assume that the market price of the European put option in (a) is 5. Which strategy would you use to lock in an arbitrage profit? Give the exact positions and the resulting payments today, in one year, and at maturity. (12 points)

(c) Calculate the value of a European call with the same characteristics as the put option in (a) using put-call-parity. (2 points)

(d) Describe the major similarities and differences between the CRR binomial model and the Black and Scholes model by filling in the following table. (8 points)

	Discrete time Binomial Model	Continuous time Model
uncertainty		
description stock price		
model		
pricing derivatives		

4. Risk-Neutral Distribution and Implied Volatility Smile (21 Points)

(a) Draw the implied volatility curve (implied volatility as a function of the strike price) that is typically observed for equity index options. Label your axis. (6 points)

(b) Describe how the Heston model improves on the Black and Scholes model in order to model underlying security processes more consistently with what is observed in real market data. How is the Heston model able to replicate implied volatility smiles and implied volatility skews. (8 points)

(c) Explain for pricing which type of options the Heston model is particularly suited. (3 points)

(d) What does the implied volatility smile in (a) imply for the shape of the risk-neutral distribution compared to the (log)normal distribution? Briefly explain the intuition. (4 points)

5. Greeks and Hedging **(23 Points)**

(a) Which problem might arise with a delta hedge if a large price movement in the underlying asset occurs? Make a suggestion how to solve that problem. (5 points)

(b) What is the delta of a European call in the Black-Scholes model? What are the bounds for this delta? (3 points)

(c) Use put-call parity to determine the delta of a European put and bounds for the put delta. (3 points)

(d) Use put-call parity to infer the relationship between the gamma of a European call and a European put option. (4 points)

(e) There exists a target option, an instrument option, an underlying stock, and a money market account with a risk-free rate of 5%. Construct a delta-gamma-neutral portfolio for hedging a long position in one target option given the information in the following table. (8 points)

Security	Delta	Gamma
Target Option (K=50, T=1)	0.85	0.035
Instrument Option (K=70, T=1.5)	0.7	0.04

6. Model Estimation (12 Points)

(a) Name two methods to calculate model parameters? Explain for each method when it is applied and how the parameters are calculated. (8 points)

(b) When using "percentage root mean squared error" as loss function for calibration of option pricing models, are OTM options assigned more, less, or equally weight than ITM options? Explain your answer in one to two sentences. (4 points)

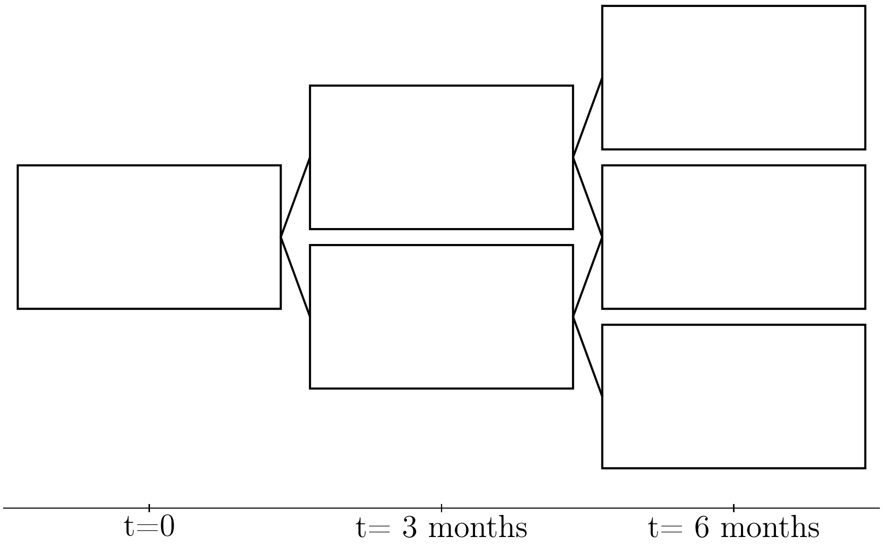
$$\%RMSE(\Theta) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left\{C_i^M - C_i(\Theta)\right\}/C_i^M\right]^2}$$

7. **Binomial Model and Exotic Options** **(24 Points)**

(a) Build a stock price tree based on the CRR model by means of the following parameters: (8 points)

- annual volatility of the stock: 0.25
- annual, continuously compounded risk-free rate: 10%
- stock price today: 80

Round all your factors to three decimal places.



(b) Determine the price of a European Lookback call that expires in 6 month ($t =$ 6 months) using risk-neutral valuation. (12 points)

(c) How does the value of a Lookback call option change as we increase the frequency with which we observe the asset price? (4 points)

Part II

Additional Exercises

1. **Option Strategies**

   Assume that you purchase a European call option with strike price K_1 and sell a call option with strike price K_2 with equal time to maturity. ($K_1 < K_2$.)

   (a) Draw the payoff diagram of your portfolio at maturity. How is this strategy called?

   (b) Assume that the stock price equals K_1 today. When would you use this strategy?

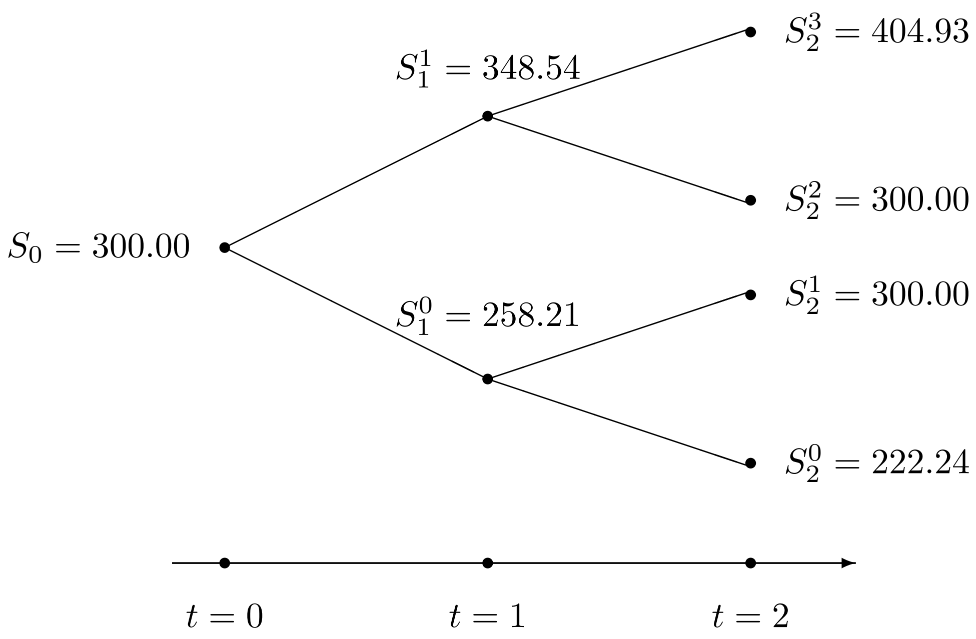
   (c) Assume that only European put options are traded in the market. How can you achieve the same payoff structure? Solve this problem graphically as well as computationally.

2. **Forwards and Arbitrage**

   The forward price of wheat for delivery in three months is \$3.90 per bushel, while the spot price is \$3.60. The three-month interest rate in continuously compounded terms is 8% per annum. Is there an arbitrage opportunity in this market if wheat may be stored costlessly? If there is an arbitrage opportunity present a way of setting up a strategy to explore the arbitrage opportunity.

3. **Binomial Tree Pricing**

   Consider a non-dividend paying stock with current price 300. The development of the stock price is described via the two-period CRR binomial tree given below where the length of one period is one year (i.e. $\Delta t = 1$). The risk-free interest rate is 7% p.a. (continuous compounding).



(a) Calculate the volatility of the stock price used to construct the binomial tree. *Hint: the tree was constructed using the same volatility factor σ in all states and at every point in time.*

(b) Use replication to determine the price of a European put with exercise price $K = 300$ that expires in one year (i.e. at $\mathbf{t} = \mathbf{1}$).
How many stocks do you have to include in the replicating portfolio at the initial point in time?

(c) Calculate the risk-neutral probabilities.

(d) Use risk-neutral pricing to determine the price of a European up-and-out call option with exercise price $K = 290$ and barrier 310, which matures in two years (i.e. at $\mathbf{t}{=}\mathbf{2}$). *Hint: If you have not solved part (3c), use a risk-neutral probability of $q{=}0.7$.*

(e) A friend of you tells you that for solving the above pricing problem he used a binomial model with more intermediate time steps and calculated a slightly different value for the option price. Explain what happens if the number of time steps increases.

4. **Bounds for Option Prices, Arbitrage**

The lower price bound of a European put (maturity date T and strike price K) on a stock with price S_t at time t is given by:

$$P_t \geq \max\{Ke^{-r(T-t)} - S_t; 0\},$$

where r denotes the risk-free interest rate and continuous compounding is used.

(a) Prove this inequality using the following two portfolios:

- Portfolio A: long European put with strike K
- Portfolio B: risk-free investment of $Ke^{-r(T-t)}$ and short position in the stock

(b) Give a possible arbitrage strategy if the inequality is violated.

5. **Hedging** **(18 Points)**

Mr. A. de Moivre works for a French bank and is very enthusiastic about the normal distribution. He believes that log returns are normally distributed and consequently uses the Black-Scholes model for option pricing. His supervisor asks him to construct a Delta-Gamma-Vega-Hedge in order to hedge a short position in a portfolio.

(a) What kind of instruments are suitable to set up a Delta-Gamma-Vega-Hedge? Explain your answer briefly. (2 points)

(b) The portfolio is already delta neutral. The gamma is -100 and the vega is -500. Mr. A. de Moivre identifies two options to construct the hedge.

	Delta	Gamma	Vega
Option 1	0.5	0.6	1.8
Option 2	-0.6	0.8	1.5

i. Give the exact position in each instrument to make the portfolio gamma and vega neutral. (5 points)

ii. What is the delta of the new portfolio (after the addition of the positions in the two options)? How can you make the portfolio delta neutral without changing the other sensitivities? (2 points)

(c) A friend approaches Mr. A. de Moivre and tells him that he does not understand Vega-Hedging.

i. Explain why the Vega of a long position in a plain vanilla option is always positive. (4 points)

ii. Comment on the statement:
"I think that Vega-Hedging makes no sense in the Black-Scholes model!"
Carefully explain why Mr. A. de Moivre should or should not engage in Vega-Hedging. (5 points)

6. **Applying Ito's Lemma**

The stock price S_t follows a geometric Brownian motion (GBM), i.e.

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

with constant parameters μ and σ.

Derive the dynamics of the following expressions and simplify the expressions as much as possible:

(a) $d(S_t^{-2})$ with $F = S_t^{-2}$

(b) $d(\ln(S_t^2))$ with $F = \ln(S_t^2)$

Now assume that X_t is governed by the following stochastic process

$$dX_t = a(b - X_t)dt + c\, dW_t,$$

where a, b, and c are positive constants, and W_t is a Wiener Process.

(c) Is the above introduced process useful to describe the behavior of stock prices?

(d) Use Ito's Lemma to determine the sde of the stochastic process of $X_t e^{at}$.

7. **Black-Scholes Pricing versus the Physical World**

Consider the following stock price processes

$$dS_t^i = \mu^i S_t^i\, dt + \sigma^i S_t^i\, dW_t^i$$

where $i = 1, 2$ and assume that dW_t^1 and dW_t^2 are uncorrelated.

(a) Compute the distribution of $\ln S_t^i$ for each i.

(b) Given the parameters $\mu^1 = 0.075$, $\mu^2 = 0.015$, $\sigma^1 = \sigma^2 = 0.1$ and $S_0^1 = S_0^2 = 100$ compute the probability that $S_T^i > 100$ for $T = 1$, for $i = 1, 2$.

(c) Consider two identical calls on each of the two stocks with $K = 100$ and $T = 1$. (Assume a complete market.) Which call would be more expensive? Explain your answer. Use the result from 7b in your explanation.

8. **Monte Carlo Simulation**

   (a) After 5000 simulation runs you obtain an Monte Carlo estimator for the option price $\widehat{C_0} = 20.05$ and a standard deviation of $\widehat{\sigma} = 10.02$. What can you say about the price estimator?

   (b) You want to reduce the variance of the estimator by means of the control variate technique. The simulation results in a value $\widehat{D_0} = 25.25$ for the control derivative that is positively correlated with the option to be priced C. The analytical value if the control derivative is $D = 25$. Given your information perform an adequate ad-hoc adjustment, assuming that $\alpha^* = 1$, to the the price estimator $\widehat{C_0}$.

   (c) Explain why the Monte Carlo simulation approach cannot easily be used for American-style derivatives.

9. **Credit Risk and Credit Derivatives**

   (a) Suppose that:

      i. The yield on a 5-year risk-free bond is 7% p.a. .

      ii. The yield on a 5-year corporate bond issued by company X is 9.5% p.a. .

      iii. A 5-year credit default swap providing insurance against company X defaulting costs 150 basis points per year.

      What arbitrage opportunity is there in this situation?

   (b) Explain how you would expect the returns offered on the various tranches in a CDO to change when the correlation between the bonds in the portfolio increases.

10. **Exotic Pricing in Continuous Time**

    Assume the Black-Scholes model holds. Consider the following derivative security with maturity T and payment structure:

$$C_T = (S_T - K) * I_{\{\overline{S} > S_T > K\}}$$

    with $K < \overline{S}$ and I denotes the indicator function

    (a) Draw the payoff of the security in a diagram. Do not forget to label axes.

    (b) How would you value this contract? Give a pricing equation.

(c) A 'pay later' option is a contract with the payoff function $C_T^{PL} = (S_T - K - \pi)I_{\{S_T \geq K\}}$. Here π denotes the premium, which is only paid if the option is ultimately exercised. Compute π such that the contract has zero value initially.

Probabilities for the standard normal distribution

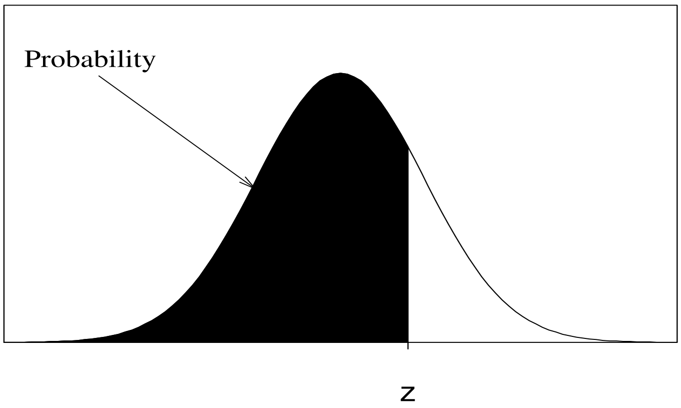


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998