

DATA STRUCTURES AND ALGORITHMS — Block 1, 2018

FINAL EXAM - RESIT

December 11, 2018

Problem 1 (25pt)

In each of the following questions, please specify if the statement is true or false. If the statement is true, explain why it is true. If it is false, explain what the correct answer is and why. (For each question, 2 points for the true/false answer and 3 points for the explanations.)

(a) (5pt) $n^{1.5} = O(n \lg n)$

(b) (5pt) A complete binary tree with depth 3 has at most 7 nodes.

(c) (5pt) $\sum_{i=1}^n 2i^3 = O(n^4)$

(d) (5pt) A chained hash table of size 64 can contain at most 64 elements.

(e) (5pt) $T(n) = 2T(n/4) + \Theta(\sqrt{n}) = \Omega(n^2)$.

Problem 2 (10pt)

(a) (6pt) Draw the binary min-heap that results from inserting 9, 3, 7, 4, 8, 2, 6, 5 in that order into an initially empty binary min-heap. You do not need to show the array representation of the heap. You are only required to show the final tree, although drawing intermediate trees may result in partial credit. If you draw intermediate trees, please circle your final result for any credit.

(b) (4pt) Draw the result of one **extract-min** call on your heap draw in part (a).

Problem 3 (8pt)

Fill in the diagram below with the result of partitioning the array with standard quicksort partitioning (taking the E at the left as the partitioning item)(6pt). Also give the number of exchanges (2pt).

E	V	E	R	Y	Q	U	A	L	K	E	Y	S	T	O	P	S	I	T

Problem 4 (10pt)

Suppose we have a max-heap H and two values v_1 and v_2 , such that all values are distinct. Let H_{12} be the heap you get if you insert v_1 and then v_2 into H , and H_{21} be the heap you get if you insert v_2 and then v_1 into H . Give an example of H , v_1 and v_2 such that $H_{12} = H_{21}$. No justification needed, just draw the heaps H , H_{12} and H_{21} .

Problem 5 (20pt)

Fill in this table with the worst-case asymptotic running time of each operation when using the data structure listed. Assume that L is a list and k is the key of an element.

	INSERT(L, k)	SEARCH(L, k)	DELETE(L, k)	getMin(L)	getMax(L)
sorted array					
unsorted array					
min-heap					
hash table with chaining					

Problem 7 (12pt)

- (a) (5pt) Draw the contents of the two open addressing hash tables in the boxes below after the following sequence of insertions: 18, 16, 10, 7, 26 ? The hash function used is $h(k) = k \bmod 9$. Assume that the probing constants (e.g. the c in $h'(k) = h(k) + ci$ for linear probing, and the equivalent two constants c_1, c_2 for quadratic probing) are all 1. Show your work for partial credit.

Linear probing

0	
1	
2	
3	
4	
5	
6	
7	
8	

Quadratic probing

0	
1	
2	
3	
4	
5	
6	
7	
8	

- (b) (3pt) What is the load factor for the tables?
- (c) (2pt) The table with linear probing will (circle one):
- (i) gradually degrade in performance as more values are inserted
 - (ii) may fail to find a location on the next insertion
 - (iii) none of the above
- (d) (2pt) The table with quadratic probing will (circle one):
- (i) gradually degrade in performance as more values are inserted
 - (ii) may fail to find a location on the next insertion
 - (iii) none of the above

Problem 7 (15pt + 5pt bonus)

Remember the *rod cutting problem* from the lecture: Given a rod of length n , and list of prices p for length i , where $1 \leq i \leq n$, how should you cut the rod into pieces in order to maximize the revenue you can get?

- (a) (10pt + 5pt bonus) Write down the bottom-up dynamic programming algorithm that solves the problem. The algorithm should return the maximum revenue r you can obtain. You can either do it in pseudocode or Python. If you write it in Python, you get bonus 5 points, but it has to run correctly. A Python algorithm that is partially correct will be considered pseudocode and will get you no bonus points.

- (b) (5pt) Draw the subproblem graph for a rod of size 4. How many vertices and edges are in the graph?