

Exam: **Sample Exam**

Code: E_EOR1_DA1

Examinator: -

Co-reader: -

Date: -

Time: -

Duration: 2 hours

Calculator allowed: Yes

Graphical calculator
allowed: No

Number of questions: 3

Type of questions: Open

Answer in: English

Credit score: 100 credits counts for a 10

Grades: The grades will be made public within 10 working days

Inspection: TBA

Number of pages: 4 (including front page)

Good luck!

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Question 1 (30/100 points)

- (a) Obtain the sample mean and median of the following data points

5.2, 2.7, 2.2, -0.7, 0.3, -1.8.

- (b) State an advantage and a disadvantage of the median compared to the sample mean as a measure of centrality.
- (c) Give the definition of a quantile of level α : q_α . For a certain variable you have obtained that the quantile of level 0.95 is $q_{0.95} = 23.5$ and the quantile of level 0.05 is $q_{0.05} = 12.7$. What can you say about the distribution of this variable?
- (d) The following R code to create a function is given

```
> oper_vec <- function(x,y){  
+   if(length(x)!=length(y)) {stop("incorrect argument")}  
+   if(!is.numeric(x)) {stop("incorrect argument")}  
+   if(!is.numeric(y)) {stop("incorrect argument")}  
+   z <- ifelse(x>y, x, y)  
+   return(z)  
+ }
```

Explain what the function “oper_vec” does (input and output).

Question 2 (40/100 points)

- (a) We have $n = 213$ observations for each of 2 variables. Consider a linear regression model of the form $y_i = \beta_0 + \beta_1 x_i + u_i$. The following sample statistics are given:

$$\bar{y} = 2.3, \quad \bar{x} = -1.3, \quad s_x^2 = 6.8, \quad s_y^2 = 11.6, \quad s_{xy} = 7.3.$$

- (i) Obtain the OLS estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.
- (ii) Obtain the R^2 . How would you interpret the value of the R^2 you have obtained?
- (iii) Obtain the TSS, RSS and ESS.
- (b) Consider the following R code to perform a regression using the data vectors y and x

```
> x3 <- x^3  
> reg <- lm(y~x+x3)
```

Write the regression model the R code is estimating. Is the intercept β_0 included or excluded from the model? How would you change the R code given above to include/exclude the intercept?

- (c) Consider the regression model given by $y_i = \beta_0 + \beta_1 x_i + u_i$. Show that the OLS estimates of β_0 and β_1 are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

by setting the partial derivatives of the sum of squares to zero.

Question 3 (30/100 points)

- (a) Discuss differences and similarities between the Maximum Likelihood (ML) rule and the ECM rule.
- (b) We have observations that take values in the set $\{0, 1, 2, 3\}$, that is, $x \in \{0, 1, 2, 3\}$. Assume we have 2 groups Π_1 and Π_2 . The distribution of Π_1 is $\text{Bin}(3, 0.5)$ and the distribution of Π_2 is $\text{Bin}(3, 0.4)$. Note that the probability mass function $f(x)$ of a binomial distribution $\text{Bin}(n, p)$ is

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad \text{for } x \in \{0, 1, \dots, n\}.$$

- (i) Draw a plot of the densities $f_1(x)$, $f_2(x)$.
- (ii) Find the sets R_1 and R_2 according to the Maximum Likelihood rule.
- (iii) Obtain the Expected Cost of Misclassification (ECM) for the ML rule when the misclassification costs are $C(1|2) = 1$ and $C(2|1) = 2$.
- (iv) Obtain the sets R_1 and R_2 of the ECM discriminant rule and the corresponding ECM.

End of the exam!