Computational Finance: Exam Questions with Answers to Technical Questions

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Exercise 1

(a). **[5 Credits]** For what purposes are options traded at financial markets?

(b). **[5 Credits]** Explain what an European call option is.

(c). **[5 Credits]** Why does it take longer computing times to price an Asian option than an European option?

(d). **[5 Credits]** What is the *volatility smile*?

(e). **[5 Credits]** What are *Greeks*?

Exercise 2

Suppose that a portfolio consists of r assets of current values x_i $(i = 1, \ldots, r)$. The value of the i-th asset next week is $x_i \, e^{Y_i}$, where $Y_1, \ldots, Y_r$ are random variables, independent, normally distributed with mean 0 and standard deviation σ_i $(i = 1, \ldots, r)$. Thus, the loss of the portfolio is

$$L = \sum_{i=1}^{r} x_i \big(1 - e^{Y_i}\big).$$

A negative loss is actually a profit. Suppose you are interested in the value of $v = \mathbb{E}[\mathbf{1}\{L > \ell\}]$ for some large threshold loss ℓ, and suppose that you apply Monte Carlo simulation for estimating v.

(a). **[5 Credits]** Give the expression of the sample average estimator using sample size n.

(b). **[5 Credits]** How do you estimate the standard error of the estimator?

(c). **[5 Credits]** How do you construct 95% confidence intervals?

(d). **[5 Credits]** Now write down the Monte Carlo algorithm for estimating b.

(e). **[5 Credits]** How would you verify your computer program (i.e, check that your code is correct)?

Exercise 3

Let $X_1(\theta), \ldots, X_n(\theta)$ be i.i.d. samples from an exponential distribution with mean θ. Suppose we sampled $n = 10$ realizations at $\theta = 2$:

$$X_1(\theta) = 1.00, X_2(\theta) = 2.13, X_3(\theta) = 0.11, X_4(\theta) = 2.12, X_5(\theta) = 6,41,$$
$$X_6(\theta) = 5.11, X_7(\theta) = 7.12, X_8(\theta) = 1.21, X_9(\theta) = 8.01, X_{10}(\theta) = 1.$$

(a.) **[5 Credits]** Compute $\frac{d}{d\theta}X(\theta)$.

(b.) **[5 Credits]** Given the above sample, what is your estimator for the 0.9 quantile of the exponential distribution with mean θ?

(c.) **[5 Credits]** Provide the IPA estimator for the quantile sensitivity with respect to θ at $\theta = 2$?

Solution:
(a). $X(\theta) = -\theta \log(1-U)$, thus

$$\frac{d}{d\theta}X(\theta) = -\log(1-U) = \frac{-\theta\log(1-U)}{\theta} = \frac{X}{\theta}.$$

(b). By $\lceil \alpha n \rceil = 9$, the estimator for the 0.9 quantile is

$$X_{\lceil \alpha n \rceil:10} = X_7(\theta) = 7.12.$$

The IPA estimator follows from (use (a.)):

$$\frac{\partial}{\partial\theta}X_{\lceil \alpha n \rceil:10} = \frac{X_7(\theta)}{\theta} = \frac{7.12}{2} = 3.06.$$

Exercise 4

[15 Credits] Let Bernoulli (θ) denote the Bernoulli distribution on $\{0, 1\}$ assigning probability θ to 1 and $1-\theta$ to 0 with density

$$f_\theta(n) = \theta^n(1-\theta)^{1-n}, \quad n \in \{0, 1\}$$

and note that $X(\theta) = \mathbf{1}\{U \leq \theta\}$ yields a Bernoulli (θ) sample, for U uniform on $[0, 1]$. Compute the Score Function for the Bernoulli-θ-distribution.

Solution: Then the Score Function reads

$$S(\theta, n) = \frac{d}{d\theta}\log f_\theta(n) = \frac{d}{d\theta}\big(n\log\theta + (1-n)\log(1-\theta)\big)$$
$$= \frac{n}{\theta} - \frac{1-n}{1-\theta}.$$

Exercise 5

A stock price is currently € 100. It is known that at the end of one month it will be either € 105 or € 95. The risk-free interest rate is zero.

(a). **[10 Credits]** What is the price of a one-month European call option with a strik4 price of € 98.

(b). **[5 Credits]** Verify your price by checking the no-arbitrage property.

(c). **[5 Credits]** How do you have to adapt your calculations if the risk-free interest rate is 2.4% per year? [Do not execute the calculations]

Solution:
(a). Sell the option for a price €C to the buyer. Then you construct a portfolio by borrowing € b at the bank, and buying Δ shares of the stock. You do this break-even, meaning $100\Delta = C + b$.

- One month later: if the stock goes up, your portfolio value becomes € $105\Delta - b$. And, you have to pay € 7 to the option buyer.
- But, if the stock goes down, the portfolio becomes € $95\Delta - b$, while the option buyer claims nothing.

Make sure that you can fullfill the claim of the buyer in both cases without making a loss or profit (no-arbitrage principle):

$$105\Delta - b = 7; \quad 95\Delta - b = 0.$$

This yields $\Delta = 0.7$. And, $b = 0.7 \times 95 = 66.5$ (euro). Thus, the option price is $C = 100\Delta - b = 3.50$ (euro).
(b). Selling for € 3.50 and borrowing € 66.50, you have € 70 for which you can buy 0.7 shares. Then, one month later, if the stock went up, your shares have value € $70 \times 1.05 =$ € 73.50. Which is exactly the amount to pay the claim € 7, and pay the loan € 66.50 back to the bank. And if the stock went down, your shares have value € $70 \times 0.95 =$ € 66.50. Which is exactly the amount to pay the loan back to the bank.
(c). The value of the portfolio one month later needs to incorporate interest rate. Annually 2.4% gives monthly 0.2%. Thus to fulfill the claim with certainty, the equations for determining Δ and b become

$$105\Delta - 1.002b = 7; \quad 95\Delta - 1.002b = 0.$$