Faculty of Science	Final Exam Complex Analysis
Department of Mathematics	26-05-2023
Vrije Universiteit Amsterdam	08:30 - 11:15

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. We consider the function $f(z) = \dfrac{\exp(z-2)}{(z-2)^2(z+2)}$, and we further assume that all closed contours below are oriented counter-clockwise.

   a) Compute $\displaystyle\int_{|z+2|=1} f(z)\,dz$ using the Cauchy integral formula. **(3P)**

   b) Determine $\displaystyle\int_{|z-2|=1} f(z)\,dz$ using the generalized Cauchy integral formula. **(3P)**

   c) Write $f(z)$ as a product of two Laurent series about $z_0 = +2$. **(5P)**

   d) Compute $\displaystyle\int_{|z|=4} f(z)\,dz$. **(2P)**

   e) Determine for each singularity of the function $g(z) = f\left(\dfrac{1}{z} + 2\right)$ its type. **(5P)**

2. We consider the function $f(z) = \dfrac{1}{z^2 - 2z + 5}$, and we further assume that all closed contours below are oriented counter-clockwise.

   a) Find the singularity of f in the upper half plane, and compute its residue. **(4P)**

   b) Compute $\displaystyle\int_{|z-i|=2} f(z)\,dz$. **(3P)**

   c) Compute the real integral $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx$. **(3P)**

3. Consider the domain $D = \{z \in \mathbb{C} : |z| < 1\}$. Are the following statements about a function f true or false? Justify your answer.

   a) If f is analytic on $D\backslash\{0\}$, then f has an anti-derivative on $D\backslash\{0\}$. **(2P)**

   b) If f has an anti-derivative on $D\backslash\{0\}$, then f is analytic on $D\backslash\{0\}$. **(2P)**

   c) If f agrees with its Maclaurin series on $D\backslash\{0\}$, then f is analytic on D. **(2P)**

   d) If the Maclaurin series of f converges on D, then f is analytic on D. **(2P)**

   <u>Hint:</u> The Maclaurin series of f is the Taylor series of f about $z_0 = 0$.

$$\text{Your grade} = \frac{\#\text{ your points}}{4} + 1$$