Faculty of Science	Midterm Exam Complex Analysis
Department of Mathematics	27-03-2023
Vrije Universiteit Amsterdam	15:30 - 17:45

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine whether the following functions defined on $\mathbb{C}$ are analytic or not.
   a) $f(z) = e^{x^2-y^2}\cos(2xy) + i \cdot e^{x^2-y^2}\sin(2xy)$ with $z = x + iy$,
   b) $f(z) = \cos(z + \overline{z}) + i \cdot \sin(z - \overline{z})$,
   c) $f(z) = \overline{\exp\left(\overline{z}^5 + 2\overline{z}^3 - 1\right)}$,

2. Compute the complex derivative $f'(z)$ of the listed analytic functions
   a) $f(z) = (x^3 - 3xy^2) + i \cdot (3x^2y - y^3)$ with $z = x + iy$,
   b) $f(z) = z^i + i^z$.

3. Find all solutions $z \in \mathbb{C}$ and write them in the form $z = x + iy$ with $x, y \in \mathbb{R}$.
   a) $4z^4 + 1 = 0$,
   b) $z - \frac{1}{3}\pi i = \mathrm{Log}(-i - \sqrt{3})$,
   c) $z - (1 - i\sqrt{3})^i = 0$,
   d) $\cos z = \frac{1}{2}\sqrt{2}$.

4. Let C denote the segment from $z_1 = -1$ to $z_2 = i$ of the unit circle. Compute the contour integral $\displaystyle\int_C z\exp(z^2)\,dz$.

5. Let C denote the positively oriented unit circle. Compute $\displaystyle\int_C \frac{\cos(z^2+2)}{z^2+4}\,dz$.

Scores:

1	2	3	4	5
a) 3	a) 3	a) 3	4	3
b) 3	b) 2	b) 3		
c) 4		c) 4		
		d) 4		
10	5	14	4	3

$$\text{Grade} = \frac{\#\text{ points}}{4} + 1$$