Vrije Universiteit Amsterdam	Complex Analysis, Final
Faculty of Sciences	25-05-2020
Department of Mathematics	15.30 - 17.30 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Check the analyticity of the following function:

$$f(x+iy) \;=\; e^x(x\cos y - y\sin y) \;+\; i\cdot e^x(x\sin y + y\cos y).$$

2. Determine all solutions $z = x+iy \in \mathbb{C}$:

   a) $z - i = (2-i)^i$,

   b) $\sin z = i$.

3. Compute the Laurent series expansion of

   a) $f(z) = \dfrac{\sin z}{(z-\pi)^2}$ around $z_0 = \pi$,

   b) $f(z) = \dfrac{z-1}{z+i}$ around $z_0 = 1$.

4. Determine for every singularity its type

   a) $f(z) = z^5 \cdot \sin(1/z^2)$,

   b) $f(z) = \dfrac{\sin(z^2)}{z^5}$.

5. Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals:

   a) $\displaystyle\int_C \cos\left(\frac{z-3}{z^2+z-12}\right) dz$

   b) $\displaystyle\int_C \frac{\exp(z^2)}{(z+1)^2}\, dz$ using the generalized Cauchy integral formula

   c) $\displaystyle\int_C \frac{\exp(z^2)}{z^2+1}\, dz$

Scoring:

1 : 3	2 : a) 3	3 : a) 4	4 : a) 3	5 : a) 3
	b) 3	b) 4	b) 3	b) 5
				c) 5
3	6	8	6	13

$$\text{Final grade} = \frac{\#\ \text{points}}{4} + 1$$