Name: Student number:

VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Exam 2
Dr. Senja Barthel	18-12-2023, 12:15-14:30 (+30 minutes extra time)

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.

You can score 33 points.

Question 1. (1+1+2+4+2 points)
Consider the function $f(x,y) = x^2 + xy$.

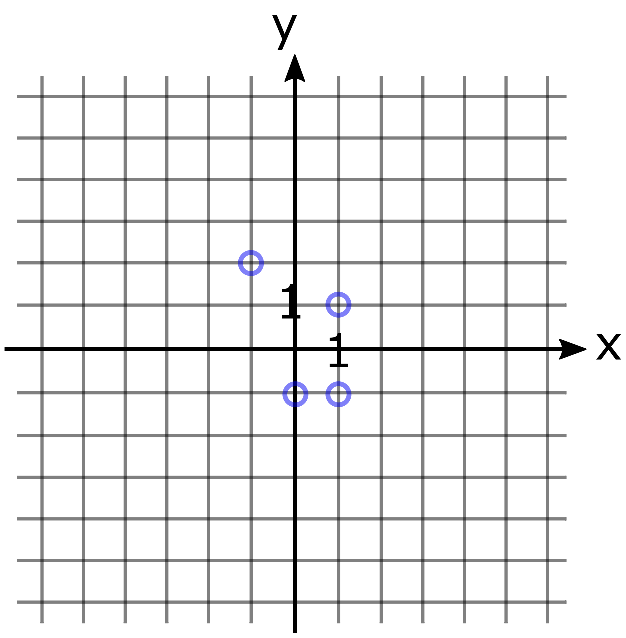
a) Compute the gradient of the function.

b) Compute the Hessian of the function.

c) Compute the directional derivative of the function in direction $v = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ at the point $p = (2,4)$.

d) Determine all maxima and minima of the function along the curve $\gamma(x) = 3x + 6$.

e) Sketch the gradient vector field of the function in the marked points $p_1 = (-1,2), p_2 = (0,-1), p_3 = (1,-1), p_4 = (1,1)$.



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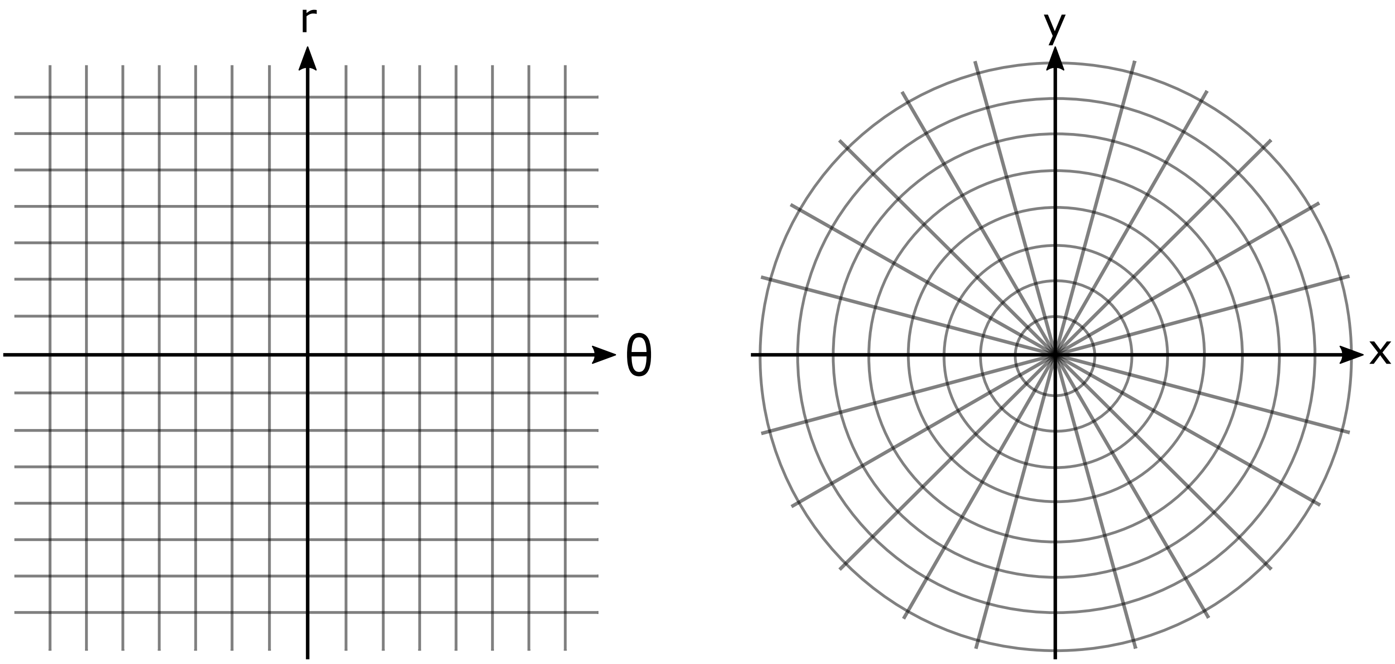
Question 2. (3+4 points)
Consider the polar curve $r = f(\theta) = 1 + \cos(\theta)$.

a) Rewrite the equation $r = 1+\cos(\theta)$ in Cartesian coordinates. (I.e., all r and θ should be replaced by x and y.) You do not need to simplify the solution.

b) Sketch the polar curve in both, the Polar and the Cartesian coordinate system. Do not forget to label the axes to indicate the step sizes you use.



Question 3. (4 points)

Compute $\displaystyle\int_0^2 \int_0^y y^2 e^{xy} dx\, dy.$

Question 4. (2 points)
What is the geometric meaning of the Jacobian determinant of a coordinate transformation (such as from Cartesian to polar coordinates or vice versa)?

Question 5. (2+3 points)

a) Compute the real part and the imaginary part of $(a + ib)^2$.

b) Explain how real numbers relate to their complex conjugates.

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Question 6. (1+4 points)

a) State the general form of a linear ordinary nonhomogeneous differential equation of order n.

b) Let $y_1(x)$ be a solution to a linear ordinary nonhomogeneous differential equation of order n, and let $y_0(x)$ be a solution to the corresponding homogeneous ordinary differential equation.
Show that the function $y(x) = y_1(x) + y_0(x)$ is also a solution of the same linear nonhomogeneous differential equation.

End of exam.