VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Exam 1
Dr. Senja Barthel	22-11-2023, 18:45-21:00 (+30 minutes extra time)

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.

You can score 36 points.

Question 1. (2 points)

Define what it means for a sequence $\{a_n\}$ to converge towards a limit L.

Question 2. (4 points)

Consider the sequence

$$a_1 = 1,\ a_2 = -2,\ a_3 = 1,\ a_n = -a_{n-3} - a_{n-2} - a_{n-1} \text{ for all } n > 3.$$

Determine whether this sequence is

a) increasing, decreasing, alternating, or none of the previous,

b) bounded (above and/or below),

c) convergent or divergent.

Question 3. (5 points)

Find the series representation on an interval including $x = 0$ for the function

$$f(x) = \frac{1 + x^3}{1 + x^2}.$$

For what values of x is the representation valid?

Question 4. (6 points)

(+1 point for each correct answer, -1 point for each wrong answer.
If the sum of all points is negative, the question is graded with zero points.)

Tick all true statements. Read carefully.

- ◯ Every conditionally convergent series is alternating.
- ◯ Every alternating series is conditionally convergent.
- ◯ There exists an alternating series that is absolutely convergent.
- ◯ Reordering a conditionally convergent series can make it absolutely convergent.
- ◯ Every convergent negative series is absolutely convergent.
- ◯ Every convergent positive series is absolutely convergent.

Turn page

Question 5. (3+2 points)

Consider the function $f(x, y) = x + 2y$.

a) Make a contour plot for the function by drawing the seven isolevels for the values $-3, -2, -1, 0, 1, 2, 3$.

b) Give a formula describing the geometric object that is defined by intersecting the graph of $f(x, y)$ with the xz-plane. What kind of object is this?

Question 6. (1+1+2 points)

a) Give the definition of the first partial derivative $f_1(x, y)$ of a differentiable function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$.

b) Describe the geometric meaning of the first derivative $f_1(a, b)$ taken in the point (a, b).

c) Compute the partial derivatives of the function $f(x, y) = 3x^2\sqrt{y}$.

Question 7. (5 points)

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be three vectors in $\mathbb{R}^3$ and t be a real value. Verify the identity

$$\boldsymbol{u} \bullet (\boldsymbol{v} \times t\boldsymbol{w}) = t\boldsymbol{v} \bullet (\boldsymbol{w} \times \boldsymbol{u}).$$

Question 8. (5 points)

Find the equation of the plane that

a) contains the line of intersection of the two planes $2x+3y-z = 0$ and $x-4y+2z = -5$,

b) and passes through the point with coordinates $(-2, 0, -1)$.

End of exam.