VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Practice questions for Exam 2
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The use of a calculator, the book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but write calculations and explain your steps.

Question 1: (3 points)
The theorem of Schwarz states that if

(a) two mixed n-th order partial derivatives of a function f involve the same differentiations but in different order

(b) and if all those partials are continuous at a point P

(c) and all partials of f of order less than n are continuous in a neighbourhood of P

Then the two partials are equal at the point P.

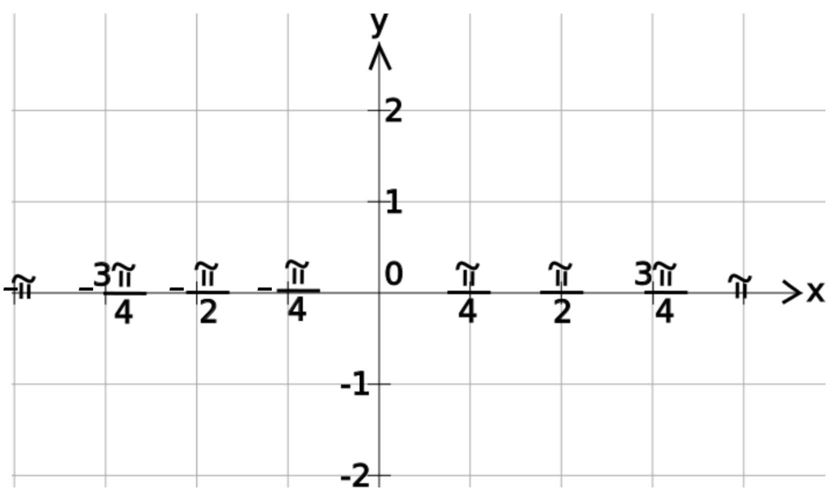
Explain how the proof of Schwarz' theorem uses (c).

Question 2: (5 points)
Let $f(x, y) = x + \sin x$ be a function from R^2 to R.

(a) Compute the gradient $\mathbf{grad}_f(a, b)$ of f in the point (a, b).

(b) Compute the directional derivative of $f(x, y)$ in the point with coordinates $(-\frac{\pi}{3}, 0.2)$, in direction of the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Question 3: (4 points)
The gradient of a function is given as the vector field $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ \cos(x) \end{bmatrix}$.
Draw the vector field by attaching a vector to each of the grid points below:



Question 4: (8 points)
Determine all points where the function $f : R^2 \to R$,
$f(x, y) = (x - 1)^3 + 3(x - 1)^2 - xy^2 - 2$ has a maximum or minimum.

Question 5: (5 points)
State for each of the following five statements whether they are true or false (here you do not need to explain your answer).
+1 point for each correct answer, -1 point for each wrong answer.
If the sum of points is negative, the question is graded with 0 points.

a) If all partial derivatives of a function exist in a point p, then all directional derivatives exist in p as well.

b) If all directional derivatives of a function exist in a point p, then all partial derivatives exist in p as well.

c) If a function is differentiable in a point p, then all partial derivatives exist in p.

d) If a function is differentiable in a point p, then all directional derivatives exist in p.

e) If all directional derivatives of a function exist in a point p, then the function is differentiable in p.

Question 6: (10 points)
Compute the volume of the object whose base is the region in the xy-plane that is bounded by the three curves $y = \mathrm{e}^x$, $y = \mathrm{e}$, $x = 0$, and the cross sections of the object parallel to the xz-plane are squares.
(Hint: It might help to sketch the object.)

Question 7: (6 points)

a) Argue that $(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$ for $n \in \mathbb{N}$, $-\pi < \theta \leq \pi$.

b) Let $z = 2 + 2i$, compute z^3.

Question 8: (8 points)
Show that $y = x$ is a solution of $x^2y'' + 21xy' - 2y = 0$ on the interval $(0, \infty)$, and find the general solution on this interval.

End of exam.