VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Practice Exam 1
Dr. Senja Barthel	Autumnn 2022, 135 minutes (+30 minutes extra time)

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.

You can score 57 points.

Question 1. (2+2+2+2 points)

For the following statements, decide whether they are true or false. If a statement is true, give an argument why that is true. If it is false, give an example that violates the statement.

a) All monotonic sequences are convergent.

b) If $\{a_n\}$ converges, then $\{\frac{1}{a_n}\}$ does not converge.

c) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n$ exists.

d) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges absolutely.

Question 2. (2+6 points)

a) State the integral test.

b) The integral test requires the involved function to be continuous and non-increasing. Explain why those conditions are needed. Can they be weakened for the theorem still to hold?

Question 3. (4+4 points)

Determine whether the following series are convergent or divergent. If the series is convergent explain if it is absolutely convergent or conditionally convergent.

a) $$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n},$$

b) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(1+n^2)}.$$

Question 4. (4+4 points)

a) Show that the Maclaurin series (Taylor series around 0) of the function $f(x) = \dfrac{3x^2}{1+x^6}$ is given by $\displaystyle\sum_{n=0}^{\infty}(-1)^n 3x^{6n+2}$.

b) Use part (a) to find the Maclaurin series representation of the function $\arctan(x^3)$.

Question 5. (3+2 points)

Consider the following three vectors

$$\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 0.5 \\ a \\ 2 \end{pmatrix}.$$

a) For which real number a do the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ span a parallelepiped of volume $V = 2$?

b) Show that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \det(\mathbf{u}, \mathbf{v}, \mathbf{w})$.

Question 6. (1+2+4+4 points)
Let P_1 and P_2 be planes and p be a point

$$\mathbf{P_1} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{P_2} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \mu_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu_2 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \qquad \mathbf{p} = (9, 2, -9).$$

a) Give the equation of P_1 in standard form $Ax + By + Cz = D$.

b) Give the equation of P_2 in normal form $(\mathbf{x} - \mathbf{v}) \cdot \mathbf{n} = 0$.

c) The planes P_1 and P_2 intersect. Give the equation of the line of intersection.

d) Compute the distance between p and the line of intersection.

Question 7. (1+3 points)

a) Sketch some of the level curves of the function $f(x, y) = \dfrac{y}{x^2 + y^2}$.

b) Determine the domain of the function.

Question 8. (5 points)
Consider the function $f(x) = x^2 - xy$. Show that the partial derivative $\frac{\partial f}{\partial x}$ obtained by using differentiation rules equals the partial derivative computed directly from the definition.

End of exam.