VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Exam 1
Dr. Senja Barthel	21-11-2022, 15:30-17:45 (+30 minutes extra time)

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.

You can score 51 points.

Question 1. (2+2+2+2 points)

For the following statements, decide whether they are true or false. If a statement is true, give an argument why that is true. If it is false, give an example that violates the statement.

a) All convergent sequences are monotonic.

b) If neither a_n nor b_n converge, then $a_n b_n$ does not converge.

c) If $a_n = 0$ for all $n \in \mathbb{N}$, then $\sum_{n=1}^{\infty} a_n$ converges.

d) If $\lim_{n\to\infty} a_n$ exists, then $\sum_{n=1}^{\infty} a_n$ converges.

Question 2. (2+3 points)

The alternating series test states that a series $\sum_{n=1}^{\infty} a_n$ converges if

(i) $a_n a_{n+1} < 0$ for all n

(ii) $|a_n| > |a_{n+1}|$ for all n

(iii) $\lim_{n\to\infty} a_n = 0$

Answer the following questions about the alternating series test:

a) Is the statement still true if $\{a_n\}$ is not alternating? Justify your answer.

b) Describe a step in the proof of the statement that relies on the second condition $|a_n| > |a_{n+1}|$.

Question 3. (2 points, 4 points, 4 points)

a) Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\frac{1}{3^n} n^2}$ convergent or divergent?

b) Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{17 + \sin(n + 5\pi)}{n^2 + 3n + 1}$ convergent or divergent?

c) For the power series $\displaystyle\sum_{k=1}^{\infty} \frac{2^k (x+1)^k}{\cos\left(\frac{1}{k}\right)}$ determine the radius and centre of convergence.

Exam continues on next page.

Question 4. (4 points)
Sketch the set of points in $\mathbb{R}^3$ that satisfy $x = 2$ and $(z-2)^2 + y^2 = 1$ and describe the set in words.

Question 5. (3+2 points)

Consider the following two vectors

$$\mathbf{u} = \begin{pmatrix} 8 \\ a \\ -2 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} a \\ 2 \\ 10 \end{pmatrix}.$$

a) For which real numbers a are the vectors $\mathbf{u}$ and $\mathbf{v}$ orthogonal with respect to the standard scalar product of $\mathbb{R}^3$?

b) Describe the geometrical meaning of $\mathbf{u} \times \mathbf{v}$.

Question 6. (3 points)

Compute the angle between the vectors $\mathbf{v} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Question 7. (1+2+4 points)

Consider the line $l = p + \lambda \mathbf{v}$, and the points A, B with

$$p = (-6, -4, 1), \qquad \mathbf{v} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}, \qquad A = (2, 2, 4), \qquad B = (-1, 6, 3).$$

a) Give an equation for the plane P that contains the line l and the point A.

b) Give an equation for the line L that is perpendicular to the plane P and that runs through the point B.

c) Compute the distance between the point B and the plane P.

Question 8. (1+4 points)

Consider the function $f(x, y) = \dfrac{x^2}{y}$.

a) Determine the domain of the function.

b) Sketch some of the level curves of the function, indicate estimate values of the function taken on the level curves.

Question 9. (2+2 points)

The continuous function $f(x, y) : R^2 \to R$ is defined as

$$f(x, y) = \begin{cases} \frac{y^3}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}.$$

Compute the partial derivatives for all points (x, y) in which the partial derivatives exist. (You can use differentiation rules as well as the definition for the computations.)

End of exam.