VU Amsterdam	Calculus 2 for BA (X_400636)
Faculty of Sciences	Second Test
dr. Gabriele Benedetti	23-12-2021, 12:15-14:30

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.
You can score 36 points. Grade=(Points/4)+1

Question 1. (4 points, 2 points)
Consider the function
$$f(x,y) = \frac{x}{y} + \ln(1 + xy^2)$$

a) Find the rate of change of f at the point $(2,1)$ in the direction of the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
Find the maximum rate of increase of f at the point $(2,1)$.

b) Compute $\frac{\partial^2 f}{\partial x \partial y}$ at the point $(2,1)$.

Question 2. (4 points, 3 points)
The function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is given by
$$f(x,y) = -xy^2 + y^3 + y^2 + \frac{x^2}{2}.$$

a) Determine all critical points of f.

b) Classify the two critical points $(0,0)$ and $(4,2)$.

Question 3. (4 points)
Use the method of Lagrange multipliers to find the minimum and maximum value of the function $f(x,y) = xy - y$ subject to the constraint $x^2 + y^2 = 1$.

Question 4. (3 points)
Compute
$$\int_0^{\sqrt[3]{\pi^2}} \int_{\sqrt{y}}^{\sqrt[3]{\pi}} 3\sin(x^3)\, dx\, dy.$$

Exam continues on next page.

Question 5. (4 points)

Let R be the finite region in the first quadrant of the xy-plane bounded by the line $y = 0$, the line $\sqrt{3}y = x$ and the curve $x^2 + y^2 = 4$. Compute

$$\int\int_R 5xy^2 \, dA.$$

Question 6. (3 points)

Transform the polar equation

$$r = \frac{1}{\sqrt{1 + 2\cos(2\theta)}}$$

to rectangular coordinates, and describe the curve represented.

Question 7. (2 points, 1 point)

a) Write the polar representation of all complex numbers z satisfying $z^3 = 2 + 2i$.

b) Compute the real and imaginary part of all complex numbers z satisfying $z^3 = 2+2i$ and belonging to the second quadrant of the complex plane.

Question 8. (3 points)

Find the function $y(x)$ solving the initial-value problem

$$\begin{cases} \dfrac{dy}{dx} = 2xe^{x^2-y}, \\ y(0) = \ln 2. \end{cases}$$

Question 9. (3 points)

Find the function $y(x)$ solving the initial-value problem

$$\begin{cases} y'' - 2y' + 5y = 0, \\ y(0) = 2, \\ y'(0) = 0. \end{cases}$$

End of exam.