VU Amsterdam	Calculus 2 for BA
Faculty of Sciences	First Test
Dr. Gabriele Benedetti	22-11-2021, 08:30-10:45

The use of a calculator, the book, or lecture notes is not permitted.
Do not just give answers, but write calculations and explain your steps.
You can score 27 points. Grade=(Points/3)+1

Question 1. (3 points)

Consider the sequence

$$\left\{ \frac{n \sin\left(n\pi - \frac{\pi}{2}\right)}{\sqrt{2n-1}} \right\}.$$

Determine whether this sequence is

a) increasing, decreasing or alternating,

b) bounded (above and/or below),

c) convergent or divergent.

Question 2. (3 points, 3 points)

Determine whether the following series are convergent or divergent. If the series is convergent, explain if it is conditionally convergent or absolutely convergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{3n}}{\ln(n+1)}$,

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-42)^n}{(n!)^2}$.

Question 3. (4 points)

Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}(1-3x)^n}{2^{n+1}}.$$

Question 4. (2 points, 1 point)

Consider the power series

$$f(x) := \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!(2n+1)},$$

which converges for all real numbers x.

a) Show that $f'(x) = e^{x^2}$.

b) Use a) to show that $\displaystyle\int_0^1 e^{x^2}\, dx = \sum_{n=0}^{\infty} \frac{1}{n!(2n+1)}$.

Exam continues on next page.

Question 5. (2 points)

Sketch the set of points in $\mathbb{R}^3$ that simultaneously satisfy the following conditions:

$$x^2 + y^2 + z^2 = 4, \quad |z| \geq 1.$$

Question 6. (1 point, 1 point, 1 point, 2 points)

The vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}, \qquad \mathbf{v} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = \mathbf{i} + a\mathbf{j} + a^2\mathbf{k},$$

where a is a real number.

a) Calculate the dot product $\mathbf{u} \bullet \mathbf{v}$ and the cross product $\mathbf{u} \times \mathbf{v}$ in terms of a.

b) Give all values of a for which $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

c) Give an equation for the plane that passes through the point $(1, 1, 3)$ and is normal to the vector $\mathbf{u}$.

d) Calculate the distance from the origin to the plane from part c).

Question 7. (2 points, 2 points)

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = y \sin\left(\frac{x}{y}\right).$$

a) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

b) Find the equation in standard form of the line that is normal to the graph of f at the point where $x = \pi$ and $y = 1$.

End of exam.