Vrije Universiteit Amsterdam	Calculus 2, Resit
Faculty of Science	06-02-2020
Department of Mathematics	18:30 - 20:30

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Explain whether the following series is conditionally convergent, absolutely convergent or divergent.

   a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{n}}{\ln n}$, b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^3}$.

2. Consider the power series
$$\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}.$$
   Determine its interval of convergence.

3. Find the Taylor series around $x = 1$ and the radius of convergence for
$$f(x) = 10^x.$$

4. The vectors $\mathbf{u}$ and $\mathbf{v}$ and point P are given by
$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}, \quad \mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ -5 \end{pmatrix} = \mathbf{j} - 5\mathbf{k} \quad \text{and} \quad P = (3, -2, 1).$$
   a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.
   b) Calculate $\mathbf{u_v}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.
   c) Give an equation for the plane passing through P and normal to the vector $\mathbf{u}$.

5. Find
$$\frac{\partial^2}{\partial y^2} f(xy^2, xy)$$
   in terms of the partial derivatives of the function f, assuming f has continuous partial derivatives of all orders.

(Please turn over)

6. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x, y) = -x^2 - xy^2 + y^2 + 6x - 2.$$

a) Determine all critical points of f.

b) Indicate for each of the critical points found in part a) if f has a local minimum value or a local maximum value, or that it is a saddle point. (If you did not reach a solution to a), you may imagine the critical points to be $(4, 1)$ and $(2, 2)$.)

c) Find the equation of the tangent plane to the graph $z = f(x, y)$ at the point $(-1, 1)$.

7. a) Compute

$$\int_0^1 \left(\int_{y\sqrt{\pi}}^{\sqrt{\pi}} \sin(x^2)\, dx \right) dy.$$

b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{ (x, y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 \leq 2 \text{ and } x \geq 0 \right\}.$$

Calculate

$$\int\int_S \frac{x}{\sqrt{x^2 + y^2}}\, dA.$$

8. Find all complex solutions z of the equation $z^4 = -16\mathrm{i}$. Express your solution(s) in the form $r(\cos(\theta) + i \sin(\theta))$.

9. Find the solution $y(x)$ to the initial value problem

$$\begin{cases} \frac{dy(x)}{dx} - xy(x) = e^{x^2/2} \sin(x), \\ y(0) = 2. \end{cases}$$

Scoring:

1 : a) 1 b) 1	2 : 3	3 : 2	4 : a) 2 b) 1 c) 1	5 : 2	6 : a) 2 b) 2 c) 1	7 : a) 2 b) 2	8 : 2	9 : 3
2	3	2	4	2	5	4	2	3

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$