VU University Amsterdam	Calculus 2, First Test
Faculty of Sciences	18-11-2019
Department of Mathematics	8.45h - 10.45h

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine whether the given sequence is (a) bounded (above and/or below), (b) increasing, decreasing or alternating, (c) convergent or divergent:

$$\left\{ \frac{n \sin n}{n^2+1} \right\}_{n=1}^{\infty}.$$

2. Determine whether the following series are convergent or divergent. If the series is convergent explain if it is conditionally convergent or absolutely convergent.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}+\sqrt{n}}$,

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{7n+(993)^n}{n!}$.

3. Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2-x)^n}{3^n n^{1/3}}.$$

4. Calculate the Taylor series in powers of $x-1$ of the function

$$f(x) = \frac{x}{1+x}$$

and determine its radius of convergence.

(Please turn over)

5. The vectors $\mathbf{u}$ and $\mathbf{v}$, and the point P are given by

$$\mathbf{u} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad P = (2, 1, 4).$$

(a) Calculate the dot product $\mathbf{u} \bullet \mathbf{v}$ and the cross product $\mathbf{u} \times \mathbf{v}$.

(b) Calculate $\mathbf{u}_{\mathbf{v}}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

(c) Give an equation for the plane passing through P which is normal to $\mathbf{u}$.

(d) Let L denote the line through P parallel to $\mathbf{v}$. What is the distance from the origin to L?

6. Consider the function

$$f(x, y) = \frac{\cos(xy)}{1 - y^2}.$$

(a) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Determine an expression for the tangent plane of f at the point $(1/2, \pi)$.

Scoring:

1 : 3	2 : (a) 3	3 : 3	4 : 3	5 : (a) 2	6 : (a) 2
	(b) 3			(b) 2	(b) 2
				(c) 2	
				(d) 2	
3	6	3	3	8	4

$$\text{Final grade} = \frac{\#\text{points}}{3} + 1$$