Vrije Universiteit Amsterdam	Calculus 2, Resit
Faculty of Science	07-02-2019
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine if the following series are convergent or divergent. If the series is convergent explain if it is conditionally convergent or absolute convergent.

$$\text{a)}\ \sum_{n=1}^{\infty} \frac{\sin(n)}{\sqrt{1+n^3}}, \qquad \text{b)}\ \sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{\sqrt{n}}.$$

2. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(2x-5)^{2n}}{n^4\, 9^n}.$$

Determine its interval of convergence.

3. a) Prove that the Maclaurin-series representation for the function

$$f(x) = \frac{e^{x^2} - e^{-x^2}}{2} \quad \text{is given by} \quad \sum_{n=0}^{\infty} \frac{x^{4n+2}}{(2n+1)!}$$

and determine for what values of x the representation is valid.

b) Use part a) to calculate the sum of the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}.$$

[Hint: consider $f'(x)$.]

4. The vectors $\mathbf{u}$ and $\mathbf{v}$ and point P are given by

$$\mathbf{u} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} = 3\mathbf{i} - \mathbf{k}, \quad \mathbf{v} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = 2\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad P = (1, 2, 3).$$

a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

b) Calculate $\mathbf{u_v}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

c) Give an equation of the plane passing through P and normal to the vector $\mathbf{u}$.

(Please turn over)

5. The function $f : \mathbb{R}^2 \to \mathbb{R}$ and the vector $\mathbf{u}$ are given by

$$f(x,y) = 2x^3 - 30x + 6xy + 3y^2 + 6y + 6 \quad \text{and} \quad \mathbf{u} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} = 3\mathbf{i} - 4\mathbf{j}.$$

a) Determine all critical points of f.

b) Indicate for each of the critical points found in part a) if f has a local minimum value or a local maximum value, or that it is a saddle point.

c) Find the directional derivative of f in $(1,2)$ in the direction of the vector $\mathbf{u}$.

6. a) Calculate the iterated integral

$$\int_0^1 \int_{\sqrt{x}}^1 \ln\left(1+y^3\right) dy\, dx.$$

b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{(x,y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 \leq 5 \text{ and } y \leq 0\right\}.$$

Calculate

$$\int\int_S e^{-x^2-y^2}\, dA.$$

7. For the real numbers a and b we have:

$$\frac{(2+2i)^3}{(\sqrt{3}+i)^4} = a + bi.$$

Calculate a and b (simplify as much as possible).

8. Solve the initial value problem

$$\begin{cases} x^2 y'(x) - y(x) = 1, \\ y(1) = 2. \end{cases}$$

Scoring:

1 : a) 2	2 : 3	3 : a) 3	4 : a) 2	5 : a) 3	6 : a) 3	7 : 2	8 : 3
b) 3		b) 3	b) 1	b) 2	b) 3		
			c) 2	c) 1			
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5	3	6	5	6	6	2	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$