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| VU University Amsterdam | Calculus 2, First Test |
| Faculty of Sciences | 19-11-2018 |
| Department of Mathematics | 12.00 - 14.00 h |

**The use of a calculator, a book, or lecture notes is <u>not</u> permitted.**
**Do not just give answers, but give calculations and explain your steps.**

1. Determine whether the given sequence is (a) bounded (above and/or below), (b) increasing, decreasing, or alternating (c) convergent or divergent:

$$\left\{\frac{\cos(n\pi)}{\sqrt{n+1}}\right\}_{n=1}^{\infty}.$$

2. Determine if the following series are convergent or divergent. If the series is convergent explain if it is conditionally convergent or absolute convergent.

a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n+4^n}{n!}$, b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n\ln(n)}{\sqrt{n}}$.

3. Consider the power series

$$\sum_{n=2}^{\infty}\frac{(2x+3)^n}{n(n-1)\,4^n}.$$

   a) Determine its interval of convergence.
   b) Suppose that this power series converges to the sum $f(x)$ on an open interval containing 0, that is

$$f(x)=\sum_{n=2}^{\infty}\frac{(2x+3)^n}{n(n-1)\,4^n}.$$

   Calculate $f''(0)$.

4. Calculate the Maclaurin-series (Taylor-series around 0) of the function

$$f(x)=\frac{2x}{3+x^2}.$$

Also determine the interval of convergence of this series.

**(Please turn over)**

5. The vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \qquad \text{and} \qquad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

b) Give all unit-vectors that are perpendicular to $\mathbf{u}$ and to $\mathbf{v}$.

c) Calculate $\mathbf{u_v}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

6. Give an equation of the plane that passes through the points $(1, 0, 3)$, $(2, 2, 3)$ and $(-1, 1, 2)$.

7. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \sqrt{12 + x^2 y^4}.$$

a) Calculate the first partial derivatives with respect to $x$ and $y$.

b) (i) Find an equation of the tangent plane to the graph of $f$ in $P = (2, 1, 4)$.

(ii) Find an equation (vector parametric, or scalar parametric, or standard form) of the normal line to the graph of $f$ in $P = (2, 1, 4)$.

## Grading:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 1 : 3 | 2 : 6 | 3 : a) 4<br>b) 3 | 4 : 3 | 5 : a) 2<br>b) 1<br>c) 1 | 6 : 3 | 7 : a) 1<br>b) 3 |
| ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| 3 | 6 | 7 | 3 | 4 | 3 | 4 |

$$\text{Final grade} = \frac{\#\text{ points} \times 3}{10} + 1$$