Vrije Universiteit Amsterdam	Calculus 2, Resit
Faculty of Science	05-04-2018
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine if the following series are convergent or divergent. If the series is convergent explain if it is conditionally convergent or absolute convergent.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n \arctan(n)}$, b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sqrt{1+\frac{1}{n^2}}$.

2. Consider the power series
$$\sum_{n=1}^{\infty} \frac{(x+3)^{2n}}{n^2\, 4^n}.$$
   Determine its interval of convergence.

3. a) Prove that the Maclaurin-series representation for the function
$$\frac{2x}{1+x^4} \quad \text{is given by} \quad \sum_{n=0}^{\infty} 2(-1)^n x^{4n+1}.$$
   b) Determine for what values of x the representation is valid.
   c) Use part a) to find the Maclaurin-series representation for the function $\arctan(x^2)$.

4. a) Give an equation of the plane that passes through the points $(1,-1,1)$, $(0,2,3)$ and $(-2,0,-1)$.
   b) Calculate the distance from the point $(2,2,1)$ to the plane from part a).

5. Assume that f has continuous partial derivatives of all orders. Find
$$\frac{\partial}{\partial x} f(xe^y, e^x y^2) \quad \text{and} \quad \frac{\partial}{\partial y} f(xe^y, e^x y^2)$$
   in terms of the partial derivatives of the function f.

(Please turn over)

6. The function $f : \mathbb{R}^2 \to \mathbb{R}$ and the vector $\mathbf{u}$ are given by

$$f(x, y) = 3x^2 - 18x + 6xy - 2y^3 + 6y^2 + 18y - 7 \quad \text{and} \quad \mathbf{u} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} = -3\mathbf{i} + 4\mathbf{j}.$$

a) Determine all critical points of f.

b) Indicate for each of the critical points found in part a) if f has a local minimum value or a local maximum value, or that it is a saddle point.

c) Find the directional derivative of f in $(1, 2)$ in the direction of the vector $\mathbf{u}$.

7. a) Calculate the iterated integral

$$\int_0^{\sqrt{\pi}} \int_x^{\sqrt{\pi}} y^2 \cos(y^2)\, dy\, dx.$$

b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{ (x, y) \in \mathbb{R}^2 \,\middle|\, 0 \le x^2 + y^2 \le 3 \text{ and } y \ge x \ge 0 \right\}.$$

Calculate, by using polar coordinates

$$\iint_S \frac{1}{\sqrt{1 + x^2 + y^2}}\, dA.$$

8. The complex number z is given by $z = \dfrac{1}{1 - e^{i\pi/3}}$. Write z in the form $a + bi$, with $a, b \in \mathbb{R}$.

9. Solve the initial value problem

$$\begin{cases} x^2 y'(x) + y(x) = 1, \\ y(1) = 2. \end{cases}$$

Scoring:

1	2	3	4	5	6	7	8	9
1 : 4	2 : 3	3 : a) 2 b) 1 c) 3	4 : a) 3 b) 1	5 : 2	6 : a) 2 b) 2 c) 2	7 : a) 3 b) 3	8 : 2	9 : 3
4	3	6	4	2	6	6	2	3

$$\text{Final grade} = \frac{\#\ \text{points}}{4} + 1$$