Vrije Universiteit Amsterdam	Calculus 2, Second Test
Faculty of Science	19-12-2017
Department of Mathematics	08.45 - 10.45 h

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Let $D = \{(x,y) \in \mathbb{R}^2, y > 0\}$. The function $f : D \to \mathbb{R}$ and the vector $\mathbf{u}$ are given by

$$f(x,y) = \frac{e^{-(x^2/4y)}}{\sqrt{y}} \qquad \text{and} \qquad \mathbf{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \mathbf{i} + \mathbf{j}.$$

   a) Show that f satisfies the (partial differential) equation

$$\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2}.$$

   b) Find the rate of change of f at the point $(2,1)$ in the direction of the vector $\mathbf{u}$.

2. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x,y) = x^2 - 2xy^3 + 3y^2.$$

   a) Determine all critical points of f.
   b) Indicate for each of the critical points found in 2a) if f has a local minimum value or a local maximum value, or that it is a saddle point.

3. Use the method of Lagrange multipliers to find the point(s) on the surface $xy + z^2 = 4$ that are closest to the origin.

4. Calculate the iterated integral

$$\int_0^2 \int_{y^2}^4 \frac{\sqrt{x}}{1+x^2}\, dx\, dy.$$

5. Calculate the area of the region in the first quadrant bounded by the parabolas $y = x^2$ and $y = 3x^2$ and the curves $y = \frac{1}{x}$ and $y = \frac{3}{x}$.
   [Hint: use an appropriate change of variables.]

(Please turn over)

6. Find all complex solutions of the equation $z^4 + iz = 0$.

7. Transform the polar equation

$$r = \frac{2}{1 + \sin\theta}$$

to rectangular coordinates and give a sketch of the curve.

8. Let $y = y(x)$. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = x^2 y^2, \\ y(0) = 1. \end{cases}$$

9. Let $y = y(x)$. Find the general (real) solution of

$$4y'' + 4y' + y = 0.$$

Scoring:

1 : a) 2	2 : a) 2	3 : 4	4 : 3	5 : 4	6 : 3	7 : 2	8 : 3	9 : 2
b) 2	b) 3							

$$\text{Final grade} = \frac{(\#\text{ points}) * 3}{10} + 1$$