VU University Amsterdam	Calculus 2, First Test
Faculty of Sciences	20-11-2017
Department of Mathematics	12.00 - 14.00 h

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine whether the given sequence is (a) bounded (above and/or below), (b) increasing or decreasing, (c) convergent or divergent:
$$\left\{\frac{n^2}{e^n}\right\}_{n=2}^{\infty}.$$

2. Calculate the sum of the (telescoping) series
$$\sum_{n=5}^{\infty} \frac{1}{n^2-7n+12}.$$

3. Determine if the following series are convergent or divergent.
$$\text{a) } \sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{2n^2-n+3}, \qquad \text{b) } \sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}.$$

4. Consider the power series
$$\sum_{n=1}^{\infty} \frac{2^n(4-x)^n}{\sqrt{n}}.$$
Determine its interval of convergence.

5. a) Prove that the Maclaurin series of the function $f(x) = x^3\cos(2x)$ is given by
$$\sum_{n=0}^{\infty} \frac{(-4)^n x^{2n+3}}{(2n)!}.$$
b) Use part a) to calculate the sum of the series
$$\sum_{n=0}^{\infty} \frac{(-1)^n(2n+3)\pi^{2n}}{(2n)!}.$$

(Please turn over)

6. The vectors $\mathbf{u}$ and $\mathbf{v}$ and point P are given by

$$\mathbf{u} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} = 3\mathbf{j} - \mathbf{k}, \qquad \mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = 2\mathbf{i} + 2\mathbf{k} \qquad \text{and} \qquad P = (-1, 3, 1).$$

a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

b) Calculate $\mathbf{u_v}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

c) Give an equation of the plane passing through P and normal to the vector $\mathbf{u}$.

d) Calculate the distance from the point $(1, 2, 3)$ to the plane from part c).

7. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \frac{x^2 + y^2}{x^2 + 1}.$$

a) Sketch the level curves $f(x, y) = c$ for $c = 0, 1$ and 2.

b) Calculate the first partial derivatives with respect to x and y.

c) Find an equation of the tangent plane to the graph of f in the point where $(x, y) = (1, 0)$.

Scoring:

1 : 3	2 : 3	3 : a) 3 b) 3	4 : 5	5 : a) 2 b) 3	6 : a) 2 b) 2 c) 2 d) 2	7 : a) 2 b) 2 c) 2
3	3	6	5	5	8	6

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$