Vrije Universiteit Amsterdam	Calculus 2, Resit
Faculty of Sciences	16-02-2017
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine if the following series are convergent or divergent.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{n^2}}{n!}$, b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$.

2. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n\,4^n}.$$

   a) Determine its interval of convergence.

   b) Suppose that this power series converges to the sum $f(x)$ on an open interval containing 0, that is

$$f(x) = \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n\,4^n}.$$

   Calculate $f'(0)$.

3. Calculate the Maclaurin-series (Taylor-series around 0) of the function

$$f(x) = \frac{x^2}{2-x}.$$

   Also determine the interval of convergence of this series.

4. The vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = -\mathbf{i} + \mathbf{j} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = 2\mathbf{i} + 3\mathbf{k}.$$

   a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

   b) Find all unit vectors perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

5. Find

$$\frac{\partial}{\partial y} f(x^2 y, xy^2)$$

   in terms of the partial derivatives of the function f, assuming that f has continuous partial derivatives of all orders.

(Please turn over)

6. The function $f : \mathbb{R}^2 \to \mathbb{R}$ and the vector $\mathbf{u}$ are given by

$$f(x, y) = x^3 - 6xy + y^3 \qquad \text{and} \qquad \mathbf{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \mathbf{i} + 3\mathbf{j}.$$

   a) Determine all critical points of f.

   b) Indicate for each of the critical points found in part a) if f has a local minimum value or a local maximum value, or that it is a saddle point.

   c) Find the directional derivative of f in $(1, 1)$ in the direction of the vector $\mathbf{u}$.

7. a) Calculate the iterated integral

$$\int_0^2 \int_y^2 \frac{x^2}{1 + x^4}\, dx\, dy.$$

   b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{ (x, y) \in \mathbb{R}^2 \,\middle|\, 4 \leq x^2 + y^2 \leq 9 \text{ and } x \leq 0 \text{ and } y \geq 0 \right\}.$$

   Calculate, by using polar coordinates

$$\int\int_S \cos\left(\pi \sqrt{x^2 + y^2}\right) dA.$$

8. For the real numbers a and b we have:

$$\frac{(1 - i\sqrt{3})^9}{(\sqrt{3} + i)^6} = a + bi.$$

   Calculate a and b.

9. Solve the initial value problem

$$\begin{cases} x\dfrac{dy}{dx} - y(x) = x^2 e^{2x}, \\ y(2) = 3e^4. \end{cases}$$

Scoring:

1 : a) 2 b) 3	2 : a) 3 b) 2	3 : 3	4 : a) 2 b) 1	5 : 2	6 : a) 2.5 b) 2.5 c) 2.0	7 : a) 3 b) 3	8 : 2	9 : 3
5	5	3	3	2	7	6	2	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$