VU University Amsterdam	Calculus 2, Second Test
Faculty of Sciences	22-12-2016
Department of Mathematics	12.00 - 14.00 h

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. The function $f : \mathbb{R}^2 \to \mathbb{R}$ and the vector $\mathbf{u}$ are given by

$$f(x,y) = y^2 \sin(xy) \qquad \text{and} \qquad \mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\mathbf{i} + \mathbf{j}.$$

   a) Find the gradient of f at $(\pi, 1)$.
   b) Find the directional derivative of f at $(\pi, 1)$ in the direction of $\mathbf{u}$.

2. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by

$$f(x,y) = x^3 - 3xy + 3y^2 - 9x + 2.$$

   a) Determine all critical points of f.
   b) Indicate for each of the critical points found in part a) if it is a local minimum, a local maximum or a saddle point.

3. Use the method of Lagrange multipliers to find the maximum and minimum value of the function $f(x,y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 34$.

4. a) The region $R \subset \mathbb{R}^2$ is bounded by the line $x = 4$ and the curve $x = y^2$. Calculate

$$\int\int_R \sqrt{x}\, e^x \, dA.$$

   b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{ (x,y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 \leq 4 \text{ and } y \geq x \geq 0 \right\}.$$

   Calculate, by using polar coordinates

$$\int\int_S \frac{1}{1 + x^2 + y^2} \, dA.$$

(Please turn over)

5. a) Find modulus $|w|$ and principal argument $\operatorname{Arg}(w)$ of $w = 1 - i\sqrt{3}$.
   b) Find all complex solutions of the equation $z^3 = 8i$.

6. Transform the polar equation $r = 2\cos\theta$ to rectangular coordinates and sketch the graph of the equation.

7. Find the general solution $y(x)$ of

$$\sqrt{x}\, y'(x) - y(x) = e^{\sqrt{x}}.$$

8. Solve the initial value problem:

$$\begin{cases} y''(x) - 4y'(x) + 13y(x) = 0, \\ y(0) = 0, y'(0) = 6. \end{cases}$$

Scoring:

1 : a) 2	2 : a) 3	3 : 4	4 : a) 4	5 : a) 1	6 : 2	7 : 4	8 : 4
b) 2	b) 3		b) 4	b) 3			

$$\text{Final grade} = \frac{\#\ \text{points}}{4} + 1$$