VU University Amsterdam	Calculus 2, First Test
Faculty of Sciences	21-11-2016
Department of Mathematics	11.00 - 13.00 h

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine whether the sequence $\left\{ \dfrac{3^n + 4^n + n}{1 + 2^{2n} + n^3} \right\}$ is convergent or divergent.

2. Calculate the sum of the series
$$\sum_{n=1}^{\infty} 3^{n-1} 4^{1-2n}.$$

3. Determine if the following series are convergent or divergent.
   a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + n - 1}$.
   b) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$.
   c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.

4. Consider the power series
$$\sum_{n=2}^{\infty} \frac{(3x-1)^n}{2^n \ln(n)}.$$
   Determine its interval of convergence.

5. The Taylor series of the function $f(x) = \dfrac{x}{2 - x^2}$ about $x = 0$ is given by
$$\sum_{n=0}^{\infty} a_n x^n.$$
   a) Calculate a_n for all $n \geq 0$ and determine for which x the series converges to $f(x)$.
   b) Use part a) to calculate $f^{(7)}(0)$.

(Please turn over)

6. The vectors $\mathbf{u}$ and $\mathbf{v}$ and point P are given by

$$\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 2\mathbf{i} + \mathbf{k}, \qquad \mathbf{v} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad P = (1, 2, 3).$$

a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

b) Give an equation of the plane passing through P and normal to the vector $\mathbf{u}$.

c) Calculate the distance from the point $(0, 1, 0)$ to the plane from part b).

7. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \frac{x}{x^2 + y^2 + 1}.$$

a) Calculate the first partial derivatives with respect to x and y.

b) Find an equation of the tangent plane to the graph of f in the point where $(x, y) = (1, 1)$.

Scoring:

1 : 2	2 : 2	3 : a) 2 b) 1 c) 2	4 : 5	5 : a) 3 b) 2	6 : a) 2 b) 1 c) 2	7 : a) 1 b) 2
2	2	5	5	5	5	3

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$