Vrije Universiteit Amsterdam	Calculus 2, Resit
Faculty of Sciences	11-02-2016
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine if the following series are convergent or divergent.

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2n}{n^2 - n\sqrt{n} + 3}$.

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$.

2. Consider the power series
$$\sum_{n=1}^{\infty} \frac{(2x-3)^n}{4^n\sqrt{n}}.$$
Determine its interval of convergence.

3. Calculate the Maclaurin-series (Taylor-series around 0) of the function
$$f(x) = \frac{x}{1-2x}.$$
Also determine the interval of convergence of this series.

4. The vectors $\mathbf{u}$ and $\mathbf{v}$ are given by
$$\mathbf{u} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$
   a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.
   b) Calculate $\mathbf{u_v}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

5. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by
$$f(x, y) = x^2 - yx^2 - y^2 + 6y - 2.$$
   a) Determine all critical points of f.
   b) Indicate for each of the critical points found in part a) if f has a local minimum value or a local maximum value, or that it is a saddle point.
   c) Find the equation of the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1)$.

(Please turn over)

6. a) Calculate the iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy.$$

b) The domain $S \subset \mathbb{R}^2$ is given by

$$S = \left\{ (x,y) \in \mathbb{R}^2 \,\middle|\, x^2+y^2 \leq 4 \text{ and } -x \leq y \leq x \right\}.$$

Calculate, by using polar coordinates

$$\int\int_S \frac{1}{\sqrt{1+x^2+y^2}}\, dA.$$

7. Find all complex solutions of the equation $z^3 = -8i$.

8. Solve the initial value problem:

$$\begin{cases} \dfrac{dy}{dx} - 2xy(x) = e^{x^2}\cos(2x), \\ y(0) = 1. \end{cases}$$

Scoring:

1 : a) 3 b) 3	2 : 4	3 : 3	4 : a) 2 b) 2	5 : a) 2 b) 3 c) 2	6 : a) 3 b) 3	7 : 3	8 : 3
6	4	3	4	7	6	3	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$