VU University Amsterdam	Calculus 2, Second Test
Faculty of Sciences	17-12-2015
Department of Mathematics	12.00 - 2.00 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Find
$$\frac{\partial}{\partial x} f(xy^2, x^2y) \qquad \text{and} \qquad \frac{\partial^2}{\partial y \partial x} f(xy^2, x^2y)$$
in terms of the partial derivatives of the function f, assuming that f has continuous partial derivatives of all orders.

2. The function $f : \mathbb{R}^2 \to \mathbb{R}$ is given by
$$f(x, y) = x^2y - x^2 + y^2 - 4y + 1.$$
   a) Determine all critical points of f.
   b) Indicate for each of the critical points found in part a) if it is a local minimum, a local maximum or a saddle point.
   c) Determine the directional derivative of f in the point $(1, 2)$ in the direction of the vector $\mathbf{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

3. Find the maximum and minimum value of the function $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$, or explain why the function has no maximum and/or minimum value subject to this constraint.

4. a) Calculate the iterated integral
$$\int_0^1 \int_{y^2}^1 \sqrt{x} e^{x^2} \, dx \, dy.$$
   b) The domain $S \subset \mathbb{R}^2$ is given by
$$S = \left\{ (x, y) \in \mathbb{R}^2 \,\middle|\, x^2 + y^2 \leq 2 \text{ and } y \geq x \right\}.$$
   Calculate, by using polar coordinates
$$\int\int_S \frac{1}{1 + \sqrt{x^2 + y^2}} \, dA.$$

(Please turn over)

5. Let $z = -2 + 2\sqrt{3}\,i$ and $w = 2 - 2i$.

   a) Calculate $|z|$ and $|w|$ and the principal values of the arguments of z and w.

   b) Write $\dfrac{z^5}{w^6}$ in the form $a + bi$, with $a, b \in \mathbb{R}$

6. a) Solve the initial value problem:

$$\begin{cases} x\dfrac{dy}{dx} = (y+1)^2, \\ y(1) = 1. \end{cases}$$

   b) Find the general (real) solution $y(x)$ of

$$y'' - 6y' + 13y = 0\ .$$

Scoring:

1 : 3	2 : a) 3 b) 3 c) 2	3 : 4	4 : a) 3 b) 3	5 : a) 2 b) 2	6 : a) 3 b) 2

$$\text{Final grade} = \frac{(\#\text{ points}) * 3}{10} + 1$$