VU University Amsterdam	Calculus 2, First Test
Faculty of Sciences	16-11-2015
Department of Mathematics	1.30 pm - 3.30 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the sequence $\{s_n\}$ defined by

$$\begin{cases} s_1 = -1 \\ s_{n+1} = s_n - \frac{1}{n+1}, \quad n \geq 1 \end{cases}$$

Does this sequence converge? Motivate your answer!

2. Determine if the following series are convergent or divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+2\sqrt{n}}{3+4n}$.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+2^n}{3^n}$.

3. a) Consider the power series

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3x+1}{4}\right)^n.$$

Determine its interval of convergence.

b) Suppose that this power series converge to the sum $f(x)$ on an open interval containing 0, that is

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3x+1}{4}\right)^n.$$

Calculate $f'(0)$.

4. Calculate the Maclaurin-series and the interval of convergence of the function

$$f(x) = \frac{x^2 - 1 + e^{-x^2}}{x}.$$

(Please turn over)

5. The vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}.$$

a) Calculate the dot-product $\mathbf{u} \bullet \mathbf{v}$ and the cross-product $\mathbf{u} \times \mathbf{v}$.

b) Calculate $\mathbf{u}_{\mathbf{v}}$, the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

6. a) Give an equation of the plane that passes through the points $(1, 1, 0)$, $(0, 2, 1)$ and $(3, 2, -1)$.

b) Calculate the distance from the point $(3, 0, -4)$ to the plane from part a).

7. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 e^{3xy}$.

a) Calculate the first partial derivatives with respect to x and y.

b) Find equations of the tangent plane and the normal line to the graph of f in the point where $(x, y) = (1, 0)$.

Scoring:

1	2	3	4	5	6	7
1 : 2	2 : a) 2	3 : a) 3	4 : 3	5 : a) 2	6 : a) 2	7 : a) 1
	b) 2	b) 3		b) 2	b) 2	b) 3
2	4	6	3	4	4	4

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$