VU University Amsterdam	Calculus 1
Faculty of Sciences	Second Test
Department of Mathematics	25-10-2022, 12:15-14:30 uur

The use of a calculator, the book, formula tables or lecture notes is <u>not</u> permitted

1. Consider the function f defined by $f(x) = \arctan(x + \ln x)$ with domain $(0, +\infty)$.
   a) Prove that f is one-to-one.
   b) Determine the domain of f^{-1}.

2. On $[-\frac{\pi}{4}, +\frac{\pi}{4}]$ the function f is defined by
$$f(x) = \left|x + \frac{\pi}{6}\right| - x + \sin^2 x.$$
   a) Find the critical point and the singular point of f.
   b) Determine the absolute maximum and the absolute minimum of f.

3. Calculate $\lim\limits_{x \to \frac{\pi}{2}} \left(\sqrt{x - \frac{\pi}{2} + 1}\right)^{1/\cos x}$.

4. Find $P_2(x)$, the second Taylor polynomial of $f(x) = x \ln x$ about $x = e^2$.

(Please turn over)

5. Compute

   a) $\int_1^2 xe^{x^2+1}\,dx$,

   b) $\int_1^2 (x^2+1)e^x\,dx$.

6. Calculate

   a) $\int \dfrac{x^4-4x^2+4}{(x-2)(x+2)}\,dx$,

   b) $\int \dfrac{x+3}{(x+2)^2+1}\,dx$.

7. Is the following statement true or false? Motivate your answer.

$$\int_1^{\pi/2} \frac{1+x^2}{\cos^2 x}\,dx < \infty$$

Scoring:

1 : a) 3 b) 3	2 : a) 3 b) 4	3 : 4	4 : 4	5 : a) 3 b) 3	6 : a) 3 b) 3	7 : 3
6	7	4	4	6	6	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$