VU University Amsterdam	Calculus 1
Faculty of Sciences	Second Test
Department of Mathematics	25-10-2020, 12:15-14:30

**The use of a calculator, the book, formula tables
or lecture notes is <u>not</u> permitted**

**Please do not just give answers to the questions,
but write calculations and motivate your assertions.**

1. Consider the function $f(x) = (x + 2) \cdot e^{\frac{1}{x}}$ with domain $(2, +\infty)$.

   a) Prove that f is one-to-one.

   b) Determine the domain of f^{-1}.

2. Consider the function $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ with domain $(0, +\infty)$.

   a) Find the *local* maximum and minimum values of f and determine which of them are also *absolute*.

   b) Calculate the x-value(s) of the inflection point(s) of the curve $y = f(x)$.

3. Calculate $\displaystyle\lim_{x \to 0^+} \left(1 + \arctan(2x)\right)^{\frac{1}{x}}$.

4. Consider the function $f(x) = \ln(\cos x)$.

   a) Find the linearization $L(x)$ of $f(x)$ about $x_0 = \frac{\pi}{4}$ and use it to give an approximate value of $\ln(\cos(\frac{\pi}{5}))$.

   b) If $E_1(\frac{\pi}{5})$ denotes the resulting error, show that
   $$\left| E_1\left(\frac{\pi}{5}\right) \right| < \left(\frac{\pi}{20}\right)^2.$$

(Please turn over)

5. Compute

   a) $\displaystyle\int_0^2 2e^{-x^2} x^3\, dx,$

   b) $\displaystyle\int_1^{e^2} \frac{\ln x}{2\sqrt{x}}\, dx.$

6. Calculate

   a) $\displaystyle\int \frac{1-x}{(x+2)^2 + 2(x+2) + 2}\, dx,$

   b) $\displaystyle\int \frac{x^3 + 2}{x^2 - x}\, dx.$

7. Is the following statement true or false? Motivate your answer.

$$\int_1^2 \frac{e^{-x^2}}{2\sqrt{x-1}}\, dx = \infty$$

Scoring:

1 : a) 3	2 : a) 3	3 : 4	4 : a) 4	5 : a) 4	6 : a) 3	7 : 3
b) 2	b) 2		b) 3	b) 2	b) 3	
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5	5	4	7	6	6	3

$$\text{Final grade} = \frac{\#\ \text{points}}{4} + 1$$