VU University Amsterdam	Calculus 1
Faculty of Sciences	Second Test
Department of Mathematics	19-10-2020, 12:15-14:30 uur

The use of a calculator, the book, formula tables or lecture notes is <u>not</u> permitted

1. Consider the function f defined by $f(x) = \sin\left(\dfrac{\pi}{x+2}\right)$ with domain $(0, +\infty)$.

   a) Prove that f is one-to-one.

   b) Determine the domain of f^{-1}.

2. The function f is defined by

$$f(x) = e^{-x} \cdot \left(x^2 - 2x - 3\right).$$

   a) Find the maxima and minima of f and classify them as local or absolute.

   b) Calculate the x-values of the inflection point(s) of the curve $y = f(x)$.

3. Calculate $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x^2} - \dfrac{1}{\ln(x+1)}\right)$.

4. Find $P_2(x)$, the second Taylor polynomial of $f(x) = \sin^{-1} x$ about $x = \frac{1}{2}\sqrt{3}$.

(Please turn over)

5. Compute

   a) $\displaystyle\int_0^{+\pi/6} (\sin x)^2 (\cos x)^3 \, dx,$

   b) $\displaystyle\int_{-\pi}^{+\pi} x^2 \cos x \, dx.$

6. Calculate

   a) $\displaystyle\int \frac{x^2+3}{x(x+3)} \, dx,$

   b) $\displaystyle\int \frac{x}{x^2-2x+2} \, dx.$

7. Is the following statement true or false? Motivate your answer.

$$\int_1^{\infty} \frac{2+\sin(x^2)}{x} \, dx = \infty$$

Scoring:

1 : a) 3 b) 3	2 : a) 4 b) 3	3 : 4	4 : 4	5 : a) 3 b) 3	6 : a) 3 b) 3	7 : 3
6	7	4	4	6	6	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$