VU University Amsterdam	Calculus 1
Faculty of Sciences	First Test
Department of Mathematics	21-09-2020, 13:30-15:45 uur

The use of a calculator, the book, formula tables or lecture notes is <u>not</u> permitted

1. Consider the polynomial
$$P(x) = x^3 - 7x + 6.$$
Show that $x - 2$ is a factor of P, and then find all roots of P.

2. Calculate the following limits, or explain why they do not exist

a) $\lim\limits_{x\to\infty} \dfrac{\sqrt{6x^2 - 4x + 7}}{|3x + 2|}$,

b) $\lim\limits_{x\to 0} \dfrac{x \sin x}{\sqrt{1 + x^2} - \sqrt{1 - x^2}}$,

c) $\lim\limits_{x\to 2} (2\lfloor x \rfloor - 1)$.

3. For which real numbers a and b is the function
$$f(x) = \begin{cases} a \cdot \cos(x + \pi/3) & x \leq 0 \\ x^2 + bx + 1 & x > 0 \end{cases}$$

a) continuous in $x = 0$?

b) differentiable in $x = 0$?

4. Prove that the equation
$$\tan x + x^3 - \frac{1}{2} = 0$$
has exactly one solution in $[0, \frac{1}{4}\pi]$ by showing that

a) it has at least one solution in $[0, \frac{1}{4}\pi]$;

b) it has at most one solution in $[0, \frac{1}{4}\pi]$.

(Please turn over)

5. Consider the graph of the equation

$$3y^2 = x^2 - 2xy.$$

a) Calculate $\dfrac{dy}{dx}$ in terms of x and y.

b) Write down the equation for the tangent line to the graph in the point $P(3, 1)$.

6. Prove, using the Mean Value Theorem, that for all $0 \leq x \leq \frac{1}{4}$:

$$2\sqrt{x} - \sin x \geq x.$$

Scoring:

1 : 4	2 : a) 3 b) 4 c) 3	3 : a) 3 b) 3	4 : a) 3 b) 3	5 : a) 4 b) 2	6 : 4
4	10	6	6	6	4

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$