VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	06-01-2020
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the function
$$f(x) = \ln(2x - x^2).$$
   a) Determine the domain D_f of f.
   b) Calculate the critical point(s) of f.
   c) Is the function concave down on its domain? Explain your answer.

2. Calculate the following limits:
   a) $\displaystyle\lim_{x\to 0^+} \left(\sqrt{1+x} - \sqrt{1-x}\right) \ln(x)$,
   b) $\displaystyle\lim_{x\to 0} (x+1)^{1/\sin(x)}$.

3. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:
$$f(x) = \begin{cases} \ln\left(\cos\left(x + \frac{\pi}{4}\right)\right) & \text{if } x > 0, \\ ax + b & \text{if } x \leq 0. \end{cases}$$
   a) For which values of a and b is f continuous at 0? Explain your answer.
   b) For which values of a and b is f differentiable at 0? Explain your answer.

4. Consider the function $f(x) = \sqrt{x} - \cos(x)$ with domain $D_f = (0, \pi)$.
   a) Prove that f has an inverse function $g = f^{-1}$.
   b) Determine the domain D_g of g.

(Please turn over)

5. Use the Mean Value Theorem to prove that for all $x \geq 0$ we have
$$\ln(x^2+1) \leq x.$$

6. Consider the function
$$f(x) = x(\ln(x) - 1).$$
Find the second-order Taylor polynomial $P_2(x)$ of $f(x)$ around $x = e^2$.

7. Calculate

   a) $\displaystyle\int_0^1 \frac{\tan^{-1}(x)}{x^2+1}\,dx,$ $\qquad$ ($\tan^{-1}$ = inverse function of tan)

   b) $\displaystyle\int x^2\cos(x)\,dx,$

   c) $\displaystyle\int \frac{5}{x^2+x-6}\,dx.$

8. Determine if the following improper integral is convergent or divergent. Motivate your answer.
$$\int_1^{\infty} \frac{2+\sin(\sqrt{x})}{x^3+\ln(x)}\,dx.$$

Scoring:

1	2	3	4	5	6	7	8
1 : a) 1	2 : a) 2	3 : a) 2	4 : a) 2	5 : 3	6 : 3	7 : a) 3	8 : 3
b) 2	b) 3	b) 2	b) 1			b) 3	
c) 3						c) 3	
6	5	4	3	3	3	9	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$