Vrije Universiteit Amsterdam	Calculus 1, Resit
Faculty of Sciences	06-01-2020
Department of Mathematics	18.30 - 21.15 pm

1. a) $x \in D_f \Leftrightarrow 2x - x^2 > 0 \Leftrightarrow x \in (0,2)$ **(1P)**.

   b) $f'(x) = \dfrac{2-2x}{2x-x^2}$ **(1P)**; $f'(x) = 0 \Leftrightarrow x = 1$ **(1P)**.

   c) $f''(x) = \dfrac{-2(2x-x^2)-(2-2x)^2}{(2x-x^2)^2}$ **(1P)**; f is concave down on its domain $\Leftrightarrow$ $f''(x) < 0$ for all $x \in D_f$ **(1P)**; and the latter holds since $(2x-x^2)^2 > 0$ and $-2(2x-x^2)-(2-2x)^2 = -2(x-1)^2 - 2 < 0$ **(1P)**.

2. a) $= \lim_{x\to 0^+} \dfrac{2x \ln x}{\sqrt{1+x}+\sqrt{1-x}}$ **(1P)** $= 0$ (x dominates $\ln x$) **(1P)**

   b) $= \lim_{x\to 0} \exp\left(\dfrac{\ln(x+1)}{\sin x}\right)$ **(1P)**; since $\lim_{x\to 0} \dfrac{\ln(x+1)}{\sin x} = \lim\limits_{x\to 0} \dfrac{1}{(x+1)\cos x} =$ 1 using l'Hopital **(1P)** we get $\exp\left(\lim_{x\to 0} \dfrac{\ln(x+1)}{\sin x}\right) = e^1 = e$ **(1P)**.

3. a) $f(0) = \ln\left(\cos\frac{\pi}{4}\right) = -\frac{\ln 2}{2}$ **(1P)** and hence when $b = -\frac{\ln 2}{2}$ and a arbitrary **(1P)**.

   b) $f'(0) = -\tan\frac{\pi}{4} = -1$ **(1P)** and hence when b as before and $a = -1$ **(1P)**.

4. a) $f'(x) = \dfrac{1}{2\sqrt{x}} + \sin x$ **(1P)**; f has an inverse function since $f'(x) > 0$ for all $x \in D_f$ **(1P)**.

   b) $D_g = R_f = (-1, \sqrt{\pi}+1)$ **(1P)**.

5. For $f(x) = \ln(x^2+1)$ we have $\dfrac{\ln(x^2+1) - \ln 1}{x - 0} = f'(c)$ for some $c \in (0, x)$ **(1P)** with $f'(x) = \dfrac{2x}{x^2+1}$ **(1P)** ≤ 1 for all $x \geq 0$ (since $x^2 + 1 \geq 2x \geq 0$) **(1P)**.

6. $P_2(x) = e^2(2-1) + f'(e^2)(x - e^2) + \frac{1}{2} f''(e^2)(x - e^2)^2$ **(1P)** with $f'(e^2) = \ln(e^2) = 2$ **(1P)** and $f''(e^2) = e^{-2}$ **(1P)**.

7. a) $\displaystyle\int \frac{\tan^{-1} x}{x^2+1}\,dx = \int u\,du$ for $u = \tan^{-1} x$ **(1P)** which equals $\frac{1}{2}u^2 = \frac{1}{2}\left(\tan^{-1} x\right)^2$ **(1P)** so that $\displaystyle\int_0^1 \frac{\tan^{-1} x}{x^2+1}\,dx = \frac{1}{2}\left(\tan^{-1} x\right)^2\Big|_0^1 = \frac{\pi^2}{32}$ **(1P)**.

   b) $= x^2 \sin x - \displaystyle\int 2x \sin x\,dx$ **(1P)** $= x^2 \sin x + 2x\cos x - \displaystyle\int 2\cos x\,dx$ **(1P)** $=$ $x^2 \sin x + 2x \cos x - 2\sin x$ **(1P)**.

   c) $\dfrac{5}{x^2+x-6} = \dfrac{A}{x-2} + \dfrac{B}{x+3}$ **(1P)** with $A = 1$ and $B = -1$ **(1P)** and hence $\displaystyle\int \frac{5}{x^2+x-6}\,dx = \ln|x-2| - \ln|x+3|$ **(1P)**.

8. $\leq 3 \displaystyle\int_1^\infty \frac{dx}{x^3 + \ln x}$ **(1P)** $\leq 3 \displaystyle\int_1^\infty \frac{dx}{x^3}$ **(1P)** $< \infty$ **(1P)**.

Scoring:

1 : a) 1	2 : a) 2	3 : a) 2	4 : a) 2	5 : 3	6 : 3	7 : a) 3	8 : 3
b) 2	b) 3	b) 2	b) 1			b) 3	
c) 3						c) 3	
6	5	4	3	3	3	9	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$