The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. The function f is defined by

$$f(x) = x^3 \left(\ln x - 1 \right).$$

    a) Determine $\lim_{x \to 0+} f(x)$ and $\lim_{x \to \infty} f(x)$.

    b) Find the extreme values of f on the interval $[1, e]$ and classify them as local or absolute.

    c) Calculate the inflection point(s) of the curve $y = f(x)$.

2. Consider the function f defined by

$$f(x) = x^5 + \tan^{-1} x.$$

Prove that f has an inverse function f^{-1} with domain $(-\infty, +\infty)$.

3. Calculate $\lim_{x \to 0} (\cos x)^{x^{-2}}$.

4.   a) Find $L(x)$, the linearization of $f(x) = \tan x$ about $x = \frac{\pi}{4}$.

       b) Use part a) to find an approximate value for $\tan \frac{\pi}{5}$.

       c) Show that the absolute value of the error is less than $\frac{1}{10}$.

(Please turn over)

5. Compute the integral $\displaystyle\int_1^{e^2} \frac{\ln\sqrt{x}}{\sqrt{x}}\,dx$ in two different ways:

    a) using substitution,

    b) using partial integration.

6. Determine

    a) $\displaystyle\int e^x \cos x\,dx.$

    b) $\displaystyle\int \frac{8x-14}{(x^2-x-6)(x-1)}\,dx.$

    c) $\displaystyle\int \frac{x-1}{x^2-4x+5}\,dx.$

7. Is the following statement true or false? Motivate your answer.

$$\int_0^{\frac{\pi}{2}} e^x \tan x\,dx = \infty.$$

Scoring:

1 : a) 2	2 : 3	3 : 4	4 : a) 3	5 : a) 2	6 : a) 2	7 : 3
b) 3			b) 2	b) 2	b) 3	
c) 2			c) 2		c) 3	
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7	3	4	7	4	8	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$