

**The use of a calculator, the book, formula tables
or lecture notes is not permitted**

1. Determine all x which satisfy the inequality:

- a) $\frac{6}{x-1} > x$,
- b) $|2x-3| \geq |x+3|$.

2. Calculate the following limits, if they exist

- a) $\lim_{x \rightarrow -\infty} \frac{2x^3 - 4x + 9}{|x^3 + 8|}$.
- b) $\lim_{x \rightarrow \infty} (2x - \sqrt{4x^2 - 3x + 5})$.

3. Consider the function $f : D(f) \rightarrow \mathbb{R}$ defined by:

$$f(x) = \frac{3 - \sqrt{25 - x^2}}{x - 4}.$$

- a) What is the domain $D(f)$ of f ?
- b) Does f have a removable singularity? If yes, then write down the continuous extension F of f .

4. Prove that the following equation has a solution x in $[0, 2\pi]$:

$$\cos(x) + \frac{x^2}{4\pi} = 0.$$

(Please turn over)

5. Consider the graph of the equation

$$\cos(y) = -\frac{y^2}{x}.$$

a) Calculate $\frac{dy}{dx}$ in terms of x and y .

b) Write down the equation for the tangent line to the graph in the point $P(\pi^2, \pi)$.

6. Prove, using the Mean Value Theorem, that for all $0 \leq x < \frac{\pi}{2}$:

$$\tan\left(x - \frac{x^2}{2\pi}\right) \geq \frac{x}{2}.$$

Scoring:

1 : a) 3 b) 3	2 : a) 3 b) 4	3 : a) 3 b) 4	4 : 4	5 : a) 4 b) 4	6 : 4
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6	7	7	4	8	4

$$\text{Final grade} = \frac{\# \text{ points}}{4} + 1$$