VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	07-01-2019
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the function
$$f(x) = x\sqrt{8x - 3x^2}.$$
   a) Calculate the domain D_f of f.
   b) Find the extreme value(s) of f on D_f and classify it/them as local or absolute.
   c) Is the function concave down on its domain? Explain your answer.

2. Calculate the following limits:
   a) $\lim\limits_{x\to 1} \dfrac{\sin(\pi x)\ln(x)}{(x-1)^2}$.
   b) $\lim\limits_{x\to\infty} x^2 - \sqrt{x^4 + 5x^2 + x\cos(x)}$.

3. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:
$$f(x) = \begin{cases} \sin\left(2x - \dfrac{1}{6}\pi\right) + 1 & \text{if } x > 0, \\ ax + b & \text{if } x \leq 0. \end{cases}$$
   a) For which values of a and b is f continuous at 0? Explain your answer.
   b) For which values of a and b is f differentiable at 0? Explain your answer.

4. The function $f : [0, \infty) \to \mathbb{R}$ is given by
$$f(x) = x^{\sqrt{x}}.$$
   a) Calculate $f'(x)$.
   b) Find the linearization $L(x)$ of f about $x = 1$.
   c) Is f one-to-one on $[0, \infty)$? Explain your answer.

(Please turn over)

5. Consider the function $f(x) = x^{1/5} = \sqrt[5]{x}$. Use the Mean Value Theorem on the interval $[32, 33]$ to prove that

$$2 < \sqrt[5]{33} < 2.0125.$$

6. Find the second-order Taylor polynomial $P_2(x)$ of the function $f(x) = \arctan(\sqrt{x})$ about $x = 1$.

7. Calculate

   a) $\displaystyle\int \tan(x) \ln(\cos(x))\, dx,$

   b) $\displaystyle\int_1^\infty \frac{6}{x^2 + 3x}\, dx,$

   c) $\displaystyle\int_0^1 x \ln(x+1)\, dx.$

8. Determine if the following improper integral is convergent or divergent. Motivate your answer.

$$\int_0^\infty \frac{2 - \sin(x^2)}{(1+x)\sqrt{x}}\, dx.$$

Scoring:

1 : a) 1	2 : a) 2	3 : a) 2	4 : a) 2	5 : 3	6 : 3	7 : a) 2	8 : 3
b) 3	b) 3	b) 2	b) 1			b) 3	
c) 2			c) 1			c) 3	
6	5	4	4	3	3	8	3

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$