Vrije Universiteit Amsterdam	Calculus 1, Second Test
Faculty of Sciences	22-10-2018
Department of Mathematics	12.00 - 2.00 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the function $f : [0, \infty) \to \mathbb{R}$ defined by

$$f(x) = \ln\left(1 + \sqrt{x}\right).$$

   a) Prove that f is one-to-one.

   b) Calculate the inverse function $f^{-1}(x)$ and specify its domain.

2. On $(0, e]$ the function f is defined by

$$f(x) = x(\ln(x))^2.$$

   a) Calculate $\lim\limits_{x \to 0+} f(x)$.

   b) Find $f'(x)$ and calculate the extreme values of f on $(0, e]$.
   Classify them as local or absolute.

   c) Determine the interval(s) where f is concave down.

3. Calculate $\lim\limits_{x \to 0} \left(\dfrac{1}{e^x - 1} - \dfrac{1}{x} \right)$.

4. a) Find $L(x)$, the linearization of $f(x) = \dfrac{1}{1 + 2x}$ about $x = 0$.

   b) Use part a) to find an approximate value for $\dfrac{1}{1.01}$ and show that the
   absolute value of the error is less than $0.0001 = 10^{-4}$.

(Please turn over)

5. The function $f : [0, \pi] \to \mathbb{R}$ is defined by

$$f(x) = \int_0^{\sin (x)} \arctan \left(4t^2\right) dt.$$

Calculate $f'\left(\frac{\pi}{6}\right)$.

6. Calculate

a) $\displaystyle\int \sin^2 (3x)\, dx.$

b) $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\, dx.$

c) $\displaystyle\int_1^2 \frac{3x + 2}{x^2(x + 1)}\, dx.$

7. Determine if the following integral is convergent or divergent. Motivate your answer.

$$\int_1^\infty \frac{1}{x \arctan (x^2)}\, dx.$$

Scoring:

1 : a) 1	2 : a) 1	3 : 3	4 : a) 1	5 : 2	6 : a) 2	7 : 2
b) 2	b) 3		b) 2		b) 3	
	c) 2				c) 3	
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3	6	3	3	2	8	2

$$\text{Final grade} = \frac{\#\ \text{points}}{3} + 1$$