Vrije Universiteit Amsterdam	Calculus 1, First Test
Faculty of Sciences	24-09-2018
Department of Mathematics	11.00 am - 1.00 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the polynomial

$$P(x) = x^3 - x^2 - 15x + 27.$$

Show that $x - 3$ is a factor of P and then find all roots of P.

2. Let $\theta \in [0, \frac{1}{2}\pi]$ be such that $\sin(\theta) = \frac{3}{5}$.
Calculate $\sin(\pi - \theta)$ and $\cos(\pi - \theta)$.

3. Consider the function $f : D_f \to \mathbb{R}$ defined by

$$f(x) = \sqrt{5 - x - \frac{4}{x}}.$$

a) Find the (maximal) domain D_f of f.
b) Calculate $f'(x)$. On what interval(s) is f increasing?

4. Calculate the following limits, or explain why the limit does not exist:

a) $\lim_{x \to 0} \dfrac{x}{\sqrt{3 + \sin(x)} - \sqrt{3 - \sin(x)}}$ b) $\lim_{x \to -\infty} \dfrac{\sqrt{1 + 2x + 3x^2}}{4 + 5x}$.

5. Let $a, b \in \mathbb{R}$. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} a|\sin(x)| + \tan(x) & \text{if } x \neq 0, \\ b & \text{if } x = 0. \end{cases}$$

a) For what value(s) of a and b is f continuous at $x = 0$?
b) For what value(s) of a and b is f differentiable at $x = 0$?
Calculate $f'(0)$ for these values.

[Explain your answers!]

(Please turn over)

6. A curve is implicitly given by the equation

$$3x^3 - x^2y + y^3 = 9.$$

a) Find $\dfrac{dy}{dx}$ in terms of x and y.

b) Calculate the equation of the tangent line to the curve at $(x, y) = (1, 2)$.

7. Use the Mean Value Theorem to show that for all $x > 0$ we have

$$\sqrt[3]{8 + 5x} < 2 + \frac{5}{12}x.$$

Scoring:

1 : 2	2 : 2	3 : a) 2 b) 3	4 : a) 3 b) 2	5 : a) 2 b) 3	6 : a) 2 b) 2	7 : 4
2	2	5	5	5	4	4

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$