VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	05-02-2018
Department of Mathematics	12.00 - 14.45 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Calculate all x which satisfy the inequality

$$\frac{x-1}{x+2} > 3.$$

2. The function $f : (0, \infty) \to \mathbb{R}$ is defined by

$$f(x) = \frac{\ln(x)}{\sqrt{x}}.$$

   a) Calculate $\lim_{x \to 0+} f(x)$ and $\lim_{x \to \infty} f(x)$.

   b) Find the extreme value(s) of f and classify it/them as local or absolute.

   c) Calculate the inflection point(s) of the curve $y = f(x)$.

3. Calculate the following limits, or explain why the limit does not exist:

   a) $\lim_{x \to 0} \dfrac{|x^2 - 1| - 1}{\sin(x)}$.

   b) $\lim_{x \to 0} \Big(1 + \sin(2x)\Big)^{\frac{3}{x}}$.

4. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x + 2x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0. \end{cases}$$

   Use the definition of the derivative to prove that f is differentiable in $x = 0$ and calculate $f'(0)$.

5. The function $f : (0, \infty) \to \mathbb{R}$ is given by

$$f(x) = 2\sqrt{x} + \ln(x).$$

   a) Prove that f is one-to-one on $(0, \infty)$.

   b) Prove that f has an inverse function g and calculate $g'(2)$.

(Please turn over)

6. Use the Mean Value Theorem to prove that for all $x > 0$:

$$\sqrt[3]{8+5x} < 2 + \frac{5}{12}x.$$

7. Find the second-order Taylor polynomial $P_2(x)$ of the function $f(x) = x\sin(\pi x)$ about $x = -1$.

8. Calculate

   a) $\displaystyle\int_3^6 \frac{3x^2}{x^2 - x - 2}\,dx,$

   b) $\displaystyle\int_0^1 \arctan(\sqrt{x})\,dx.$

9. Determine if the following improper integral is convergent or divergent. Motivate your answer.

$$\int_0^{\pi} \frac{3+\cos(x)}{(1+x^2)\sqrt{x}}\,dx.$$

Scoring:

1 : 3	2 : a) 2 b) 3 c) 2	3 : a) 2 b) 3	4 : 3	5 : a) 2 b) 2	6 : 3	7 : 3	8 : a) 3 b) 3	9 : 2
3	7	5	3	4	3	3	6	2

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$