Vrije Universiteit Amsterdam	Calculus 1, Second Test
Faculty of Sciences	23-10-2017
Department of Mathematics	12.00 - 2.00 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. The function $f : (0, \infty) \to \mathbb{R}$ is defined by

$$f(x) = x^2 - 2x^2 \ln x.$$

   a) Calculate $\lim_{x \to 0+} f(x)$ and $\lim_{x \to \infty} f(x)$.

   b) Find the extreme values of f and classify them as local or absolute.

   c) Calculate the inflection point(s) of the curve $y = f(x)$.

2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{x^5}{x^2 + 1}.$$

   a) Prove that f has an inverse function f^{-1} <u>with domain $\mathbb{R}$</u>.

   b) Calculate $(f^{-1})'\left(\frac{1}{2}\right)$.

3. Calculate $\lim_{x \to \infty} x\,(\pi - 2 \arctan x)$.

4. a) Find $L(x)$, the linearization of $f(x) = \sqrt{x}$ about $x = 100$.

   b) Use part a) to find an approximate value for $\sqrt{102}$ and show that the absolute value of the error is less than 0.0005.

(Please turn over)

5. The function $\sinh x$, the hyperbolic sine, is defined by

$$\sinh x = \frac{e^x - e^{-x}}{2}.$$

Find the Maclaurin polynomial of order $2n+1$ ($n \in \mathbb{N}$) for $\sinh x$.
[Hint: Use the Taylor formula for e^x at $x = 0$.]

6. Calculate

a) $\displaystyle\int x \arctan x \, dx.$

b) $\displaystyle\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x} \, dx.$

c) $\displaystyle\int_3^{\infty} \frac{1}{x^2 - 4} \, dx.$

7. Determine if the following integral is convergent or divergent. Motivate your answer.

$$\int_0^{\infty} \frac{1}{(1+x^3)\sqrt{x}} \, dx.$$

Scoring:

1 : a) 2	2 : a) 3	3 : 3	4 : a) 2	5 : 3	6 : a) 3	7 : 4
b) 3	b) 2		b) 3		b) 3	
c) 2					c) 3	
7	5	3	5	3	9	4

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$