Vrije Universiteit Amsterdam	Calculus 1, First Test
Faculty of Sciences	25-09-2017
Department of Mathematics	11.00 am - 13.00 am

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine all x which satisfy the inequality

$$\frac{1}{x+1} \leq 1 + \frac{x}{2}.$$

2. Prove the given identity:

$$\frac{1-\cos x}{1+\cos x} = \tan^2\left(\frac{x}{2}\right).$$

3. Consider the function $f : D_f \to \mathbb{R}$ defined by

$$f(x) = \frac{x}{1-\sqrt{x^2+1}}.$$

a) Find the (maximal) domain D_f of f.

b) Is f an even function? Is f an odd function? [Explain your answers.]

c) On what interval(s) is f increasing? And on what interval(s) is it decreasing?

d) Calculate the following limits, or explain why the limit does not exist:

$$\lim_{x\to\infty} f(x), \quad \lim_{x\to-\infty} f(x) \quad \text{and} \quad \lim_{x\to 0} f(x).$$

e) Find the range R_f of f.

4. Calculate the following limits:

a) $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{x^2+3x}$, b) $\displaystyle\lim_{x\to 2+} \frac{|2x-x^2|}{4-x^2}$.

(Please turn over)

5. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} \tan\left(x + \frac{\pi}{3}\right) & \text{if } x \geq 0, \\ ax + b & \text{if } x < 0. \end{cases}$$

a) For which values of a and b is f continuous at $x = 0$?

b) For which values of a and b is f differentiable at $x = 0$?

[Explain your answers!]

6. A curve is implicitly given by the equation

$$x^3 - 3xy + y^3 = 1.$$

a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x and y.

b) Calculate the equation of the tangent line to the curve at $(x, y) = (1, 0)$.

Scoring:

1 : 3	2 : 2	3 : a) 1	4 : a) 2	5 : a) 2	6 : a) 3
		b) 1	b) 2	b) 3	b) 2
		c) 2			
		d) 3			
		e) 1			
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3	2	8	4	5	5

$$\text{Final grade} = \frac{\#\ \text{points}}{3} + 1$$