VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	10-01-2017
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the function
$$f(x) = \sqrt{4\sqrt{x} - x}.$$

   a) Calculate the domain of f.

   b) Calculate the extreme values of f on $[1, 9]$.

2. Calculate the following limits, or explain why the limit does not exist:

   a) $\displaystyle\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$.

   b) $\displaystyle\lim_{x \to 0} \left(1 + \tan(2x)\right)^{\frac{1}{x}}$.

3. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} \dfrac{e^{2x} - 1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

   Use the definition of derivative to prove that f is differentiable in 0 and calculate $f'(0)$.

4. Use the Mean Value Theorem to prove that for all $x > 0$
$$(1 + x)^{3/2} > 1 + \frac{3}{2}x.$$

5. A curve is implicitly given by the equation
$$(x^2 + y^2)^2 = 50xy.$$

   Find the equation of the tangent line to the curve at $(x, y) = (2, 4)$.

(Please turn over)

6. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + e^x$.

   a) Prove that f has an inverse function f^{-1} with domain $\mathbb{R}$.

   b) Calculate $(f^{-1})'(1)$.

7. Find the second-order Taylor polynomial of $\arctan x$ about $x = 1$.

8. Calculate

   a) $\displaystyle\int \sqrt{x}\ln x\,dx$,

   b) $\displaystyle\int_0^1 \frac{1}{e^x + 1}\,dx$,

   c) $\displaystyle\int_{-2}^{\infty} \frac{1}{x^2 + 4x + 8}\,dx$.

9. Determine if the following integral is convergent or divergent. Motivate your answer.

$$\int_1^{\infty} \frac{\arctan x}{x\sqrt{x}}\,dx.$$

Scoring:

1 : a) 1	2 : a) 3	3 : 3	4 : 3	5 : 3	6 : a) 2	7 : 3	8 : a) 2	9 : 2
b) 3	b) 2				b) 2		b) 4	
							c) 3	
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4	5	3	3	3	4	3	9	2

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$