Vrije Universiteit Amsterdam	Calculus 1, Second Test
Faculty of Sciences	24-10-2016
Department of Mathematics	12.00 - 2.00 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = x^2 e^{-2x}.$$

   a) Find the extreme values of f and classify them as local or absolute.

   b) Calculate the x-values of the inflection point(s) of the curve $y = f(x)$.

2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = (1 + x^2) \arctan(x).$$

   a) Prove that f has an inverse function f^{-1} with domain $\mathbb{R}$.

   b) Calculate $(f^{-1})'(\pi/2)$.

3. Calculate $\lim_{x\to\infty} \left(e^{2x} + 3x\right)^{\frac{1}{4x}}$.

4. a) Find $P_3(x)$, the third-order Taylor polynomial of $f(x) = \sin(\pi x)$ about $x = 1$.

   b) Use part a) to calculate the limit

$$\lim_{x\to 1} \frac{\sin(\pi x) + \pi x - \pi}{(x-1)^3}.$$

(Please turn over)

5. Calculate

a) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx.$

b) $\displaystyle\int_0^1 \frac{3x+2}{x^2-4}\,dx.$

6. a) Show that the integral $I_n = \int_0^{\pi/2} x^n \sin(x)\,dx$ satisfies the reduction formula

$$I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}, \qquad \text{for } n \geq 2.$$

b) Use part a) to evaluate I_4.

7. Determine if the following integral is convergent or divergent. Motivate your answer.

$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx.$$

Scoring:

1 : a) 3	2 : a) 2	3 : 3	4 : a) 2	5 : a) 2	6 : a) 3	7 : 4
b) 2	b) 2		b) 2	b) 3	b) 2	
____	____	____	____	____	____	____
5	4	3	4	5	5	4

$$\text{Final grade} = \frac{\#\text{ points} \times 3}{10} + 1$$