Vrije Universiteit Amsterdam	Calculus 1, First Test
Faculty of Sciences	26-09-2016
Department of Mathematics	11.00 am - 13.00 am

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine all x which satisfy the inequality

$$\frac{x}{3} \leq 1 + \frac{2}{x+2}.$$

2. Let $\theta \in [0, \pi]$ be such that $\tan(\theta) = -\frac{3}{4}$. Calculate $\sin(\theta)$ and $\cos(\theta)$.

3. Calculate the following limits, or explain why the limit does not exist:

   a) $\lim\limits_{x \to 3} \left(\dfrac{1}{x-3} - \dfrac{6}{x^2-9} \right)$.

   b) $\lim\limits_{x \to 0} \dfrac{|x - x^2|}{\sin(x)}$.

   c) $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + 2x - 1} \right)$.

4. Explain why the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^3}\right) & \text{if } x \neq 0, \\ 3 & \text{if } x = 0, \end{cases}$$

   has a removable discontinuity at $x = 0$. To remove it, redefine $f(0)$.

(Please turn over)

5. Find values of a and b that make

$$f(x) = \begin{cases} x^2 + ax + b, & \text{if } x < 0, \\ \tan(x + \pi/4), & \text{if } x \geq 0, \end{cases}$$

differentiable at $x = 0$.

6. Prove that the equation

$$x^5 + 4x + \cos(3x) = 0$$

has exactly one real solution.
[Hint: Show that the equation has at least one solution and then show that it has at most one solution. Justify your conclusions!]

7. A curve is implicitly given by the equation

$$x \sin(y) + y^3 = 2x \cos(x).$$

Find the equation of the tangent line to the curve at $(x, y) = (\pi/2, 0)$.

Scoring:

1 : 3	2 : 3	3 : a) 2 b) 3 c) 2	4 : 3	5 : 4	6 : 4	7 : 3
3	3	7	3	4	4	3

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$