VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	05-01-2016
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Calculate the following limits, or explain why the limit does not exist:

   a) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+x^3}}{\sin(x)}$.

   b) $\lim\limits_{x\to -\infty} \left(x+\sqrt{x^2+3x+1}\right)$.

2. The function $f:\mathbb{R}\to\mathbb{R}$ is given by $f(x)=e^x\cos x$.

   a) Find the linearization $L(x)$ of f about $x=0$.
   b) Determine the minimum and maximum values of f on $[-\frac{1}{2}\pi,\frac{1}{2}\pi]$.
   c) Calculate the inflection point(s) of the curve $y=f(x)$ on $[-\frac{1}{2}\pi,\frac{1}{2}\pi]$.

3. In this exercise $c\geq 0$ is a real constant.
   The function $f_c:\mathbb{R}\to\mathbb{R}$ is given by

$$f_c(x)=\begin{cases} c-x^2 & \text{if } x\leq 0,\\ \sqrt{x^2+c} & \text{if } x>0.\end{cases}$$

   Determine for which value(s) of c the function f_c is continuous at $x=0$.

4. Consider the function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\frac{x}{\sqrt{x^2+4}}.$$

   a) Prove that f is one-to-one on $\mathbb{R}$.
   b) Prove that f has an inverse function f^{-1} with domain $(-1,1)$.
   c) Calculate $(f^{-1})'(0)$.

5. A curve is implicitly given by the equation

$$y+\cos(y)=x^2+\ln(x).$$

   Find the equation of the tangent line to the curve at $(x,y)=(1,0)$.

(Please turn over)

6. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x\sqrt{|x|}\sin(\ln|x|) & \text{if } x \neq 0. \end{cases}$$

Use the definition of derivative to prove that f is differentiable in 0 and calculate $f'(0)$.

7. Prove that the equation

$$\sin(x) + \cos(x) = 3x - 2$$

has exactly one (real) solution.
[Hint: consider the function $f(x) = \sin(x) + \cos(x) - 3x + 2$.]

8. Calculate

a) $\int e^{\sqrt{x}}\, dx.$

b) $\int_1^\infty \frac{1}{x^3 + x^2}\, dx.$

Scoring:

1 : a) 2 b) 3	2 : a) 2 b) 3 c) 2	3 : 3	4 : a) 2 b) 2 c) 1	5 : 3	6 : 3	7 : 3	8 : a) 3 b) 4
5	7	3	5	3	3	3	7

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$